1.1A – Function Concepts

Relation – a set of ordered pairs that can be represented graphically by points or in many cases by a line or curve. In the latter case, one can often come up with an algebraic model to describe the set of points.

Ex. Rolling two dice. The first dice gives the x-value and the second gives the y. Sample output might include: (1,2), (2,5), (6,2) (1,3)

Ex. $\boldsymbol{x^2 + y^2 = 9}$ (a circle with centre at origin and radius of 3)

Notice that x=1 domain matches with two different y values, so not a function

Function – a special relation in which each element of the domain (x-value) corresponds to only one element of the range (y-value). Idea is that each input value (x) will return only one unique output value (y). This is readily evident when graphed as no two points can be vertically on top of each other. A function will pass the Vertical Line Test (VLT)

Ex. The cost of a movie is based on your age. Two people of the same age cannot be charged a different amount. So tell one your age (input) and one can uniquely calculate the cost.

Ex. $m(x) = 2x^3 - 4x^2 + 3x - 1$

Function notation first used by Bernoulli (1718) differs slightly from the formula notation and provides a useful tool when evaluating a function or considering transformations and combinations of functions.

Ex. $y = 2x + 1$ corresponds to $f(x) = 2x + 1$

$y = (x - 1)^2 + 2$ corresponds to $g(x) = (x - 1)^2 + 2$

$x^2 + y^2 = 25$ (Is not a function so need to rewrite and separate)

$$m(x) = \begin{Bmatrix} +\sqrt{25 - x^2} \\ -\sqrt{25 - x^2} \end{Bmatrix}$$

(Top of circle: $+\sqrt{25-x^2}$; Bottom: $-\sqrt{25-x^2}$)

Evaluating a function is best done using function notation. This tells one the value of the function (y-value) for a given input (x-value). Using the technique repeatedly in a table of sequential values will generate a set of points that can be joined to form a curve.

Ex. Given $f(x) = 3x + 1$ then evaluating at $x = 5$ $f(5) = 3(5) + 1$
$= 16$

Composite functions are made when two or more functions are combined. Some specific notation helps one keep track of these combining steps.

Ex. Given two functions *f(x)* and *g(x)*

$f \circ g$ means $f[g(x)]$

$g \circ f$ means $g[f(x)]$

Inverting a function implies undoing what was done. That is one wants to know how to start with the functions output (y-value) and get back to the input (x-value). In doing so, one often, but not always, creates a new function – the inverse function to the original. Again specific notation is used.

Ex. If $g(x) = 2x - 3$ then its inverse $g^{-1}(x) = (x + 3) \div 2$

-1 is not an exponent in this case but is used for inverse notation

Example 1: Evaluate the following functions at the stated value.

a) $f(x) = 2x - 1$ at x = 3

$$f(3) = 2(3) - 1$$
$$= 5$$

b) $g(x) = 3x^4 - \sqrt[3]{25 + x}$ at x = 2

$$g(2) = 3(2)^4 - \sqrt[3]{25 + 2}$$
$$= 48 - 3$$
$$= 45$$

Example 2: Find the inverse of the following functions.

a) $h(x) = \sqrt{x - 1}$

Easiest to switch x and y in formula form

$$x = \sqrt{y - 1}$$
$$x^2 = y - 1$$

Rearrange for y = ?

$$x^2 + 1 = y$$
$$\therefore h^{-1}(x) = x^2 + 1$$

Lastly re-write in function notation

b) $g(x) = 3x^2 + 5$

$$x = 3y^2 + 5$$
$$x - 5 = 3y^2$$
$$\frac{x - 5}{3} = y^2$$
$$\sqrt{\frac{x - 5}{3}} = y$$
$$\therefore g^{-1}(x) = \sqrt{\frac{x - 5}{3}}$$

Example 3: Given the functions listed below, evaluated the following;

$$f(x) = 2x - 3 \qquad g(x) = (x - 3)^2 + 1 \qquad m(x) = \frac{x - 1}{2}$$

a) $f(3) = ?$

$$f(3) = 2(3) - 3$$
$$= 3$$

b) $g(-2) = ?$

$$g(-2) = [(-2) - 3]^2 + 1$$
$$= (-5)^2 + 1$$
$$= 26$$

c) $g \circ f(1) = ?$

$$g \circ f(1) = g[f(1)]$$
$$= g[2(1) - 3]$$
$$= g[-1]$$
$$= [(-1) - 3]^2 + 1$$
$$= [-4]^2 + 1$$
$$= 17$$

Or one could find g[f(x)] 1st and then substitute into it.

d) $m^{-1}(3) = ?$

Find inverse function first

$$x = \frac{y - 1}{2}$$
$$2x + 1 = y$$
$$so\ m^{-1}(x) = 2x + 1$$
$$m^{-1}(3) = 2(3) + 1$$
$$= 7$$

1.1A – Function Concepts Practice Questions

1. Evaluate the following functions at for the value given.

a) $f(x) = 2x - 1$ at x = 7
b) $g(x) = 3x - 2$ at x = -2
c) $h(x) = (x-1)^2 + 2$ at x = 4
d) $m(x) = 2\sqrt{x+3} - 1$ at x = 1
e) $y = 2x^3 - 1$ at x = -5
f) $y = \sqrt[3]{x+2} + 5$ at x = 6

2. Determine the inverse of the following functions;

a) $h(x) = \sqrt{x-1}$
b) $g(x) = 3x^2 + 5$

3. Given the functions listed below, write a simplified expression for the following;

$$f(x) = 2x - 3 \qquad g(x) = (x-3)^2 + 1 \qquad m(x) = \frac{x-1}{2}$$

a) *f o g*
b) *g o f*
c) *f[m(x)]*
d) g^{-1}*[f(x)]*

4. Use the functions listed in above question (#3) to evaluate the following;

a) f(-5)
b) g(2)
c) m(3)
d) *g o f (2)*
e) *f o g (1)*
f) $m^{-1}(7)$
g) *f o g o m (5)*
h) g^{-1}[f(m(6))]

5. If *h* and *k* are linear functions, show the following.

a) *h o k* is also a linear function
b) In general *h o k* ≠ *k o h*
c) Give an example of linear functions for which *h o k = k o h*

6. Use a table of values to evaluate the following functions repeatedly at various sequential test values and then come up with a rough sketch of the following functions. Comment on any domain areas that present difficulty and why. Check your sketches with graphing calculator.

a) $f(x) = 2(x-4)^2 + 1$
b) $g(x) = \frac{1}{x^2 - 4}$
c) $h(x) = \sqrt{x-4}$

Answers 1. a) 13 **b)** -8 **c)** 11 **d)** 3 **e)** -251 **f)** 7 **2. a)** $h^{-1}(x) = x^2 + 1$ **b)** $g^{-1}(x) = \sqrt{\frac{x-5}{3}}$ **3. a)** $2(x-3)^2-1$ **b)** $(2x-6)^2+1$ **c)** x-4 **d)** $\sqrt{2x-4} + 3$ **4. a)** -13 **b)** 2 **c)** 1 **d)** 5 **e)** 7 **f)** 15 **g)** 1 **h)** 4 **5. c)** any lines through origin