1.1B – Function Concepts Graphical Properties

Zeros – the domain (x-value) location(s) where the function (y-value) equals zero. Since the x-axis has the value of y = 0 this can also be thought of as x-value location(s) where the graph crosses or touches the x-axis.

Ex. Graphically one could use a calculator to see approximate x-value(s)
Ex. Algebraically one could set the function equal to zero and solve the resulting equation to get exact x-value(s) when possible

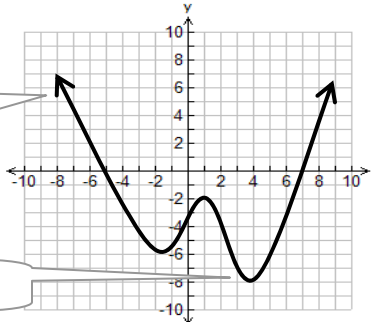
Increasing and Decreasing intervals talk about whether the function increases or decreases it value over a given domain.

Ex. An increasing function will have a positive slope over the considered domain. Graphically, this basically means as one moves from left to right the value of the function will increase (i.e. goes from $f(x) = 3$ to $f(x) = 4$)
Ex. A decreasing function will show a negative slope. Algebraically one could check two consecutive domain (x) values to see function has a lower value.

Maximum or Minimum values of a function occur locally (relatively) at peaks (max) or valleys (min). One is interested in stating the function output (y-value) at the given domain input. For the absolute max/min of a function one considers all possible domain values to see the highest (max) or lowest (min) value the graph ever reaches.

Example 1: Graphically state the zeros, increasing/decreasing intervals, local (relative) max/min and absolute maximum or minima;

a)



No max as function goes up forever

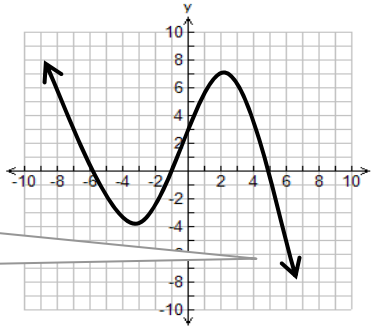
Lowest function ever goes

Zeros at: x = -5 and + 7
Increasing intervals: -2 < x < +1 & +4 < x < ∞
Decreasing intervals: -∞ < x < -2 & +1 < x < +4
Local Maximum(s): y = -2 occurs when x = 1
Local Minimum(s): y = -6 & y = -8
Absolute maximum: none
Absolute minimum: y = - 8

Note inequality direction

infinity

b)



No min as function goes down forever

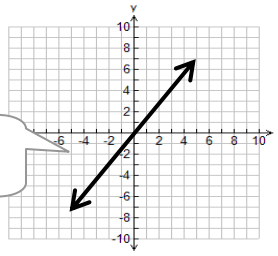
Zeros at: x = -6, -1, and +5
Increasing intervals: -3 < x < +2
Decreasing intervals: x < -3 & +2 < x
Local Maximum(s): y = +7
Local Minimum(s): y = -4
Absolute maximum: none
Absolute minimum: none

Do not need to write infinity symbol as is implied by statement

Example 2: The next page outlines some of the more common functions we will need to know this year. Note the basic shapes and some of the key points and features of these functions.

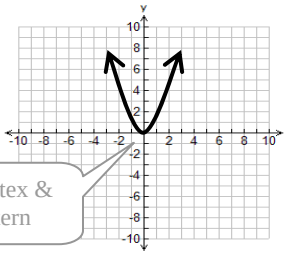
Polynomial functions

$f(x) = x$ (linear)



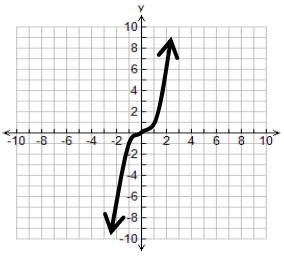
Intercept & Slope

$g(x) = x^2$ (quadratic)



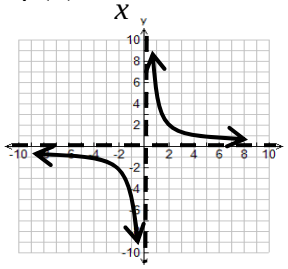
Vertex & Pattern

$m(x) = x^3$ (cubic)

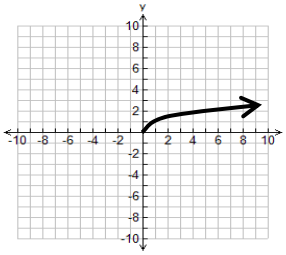


Rational functions

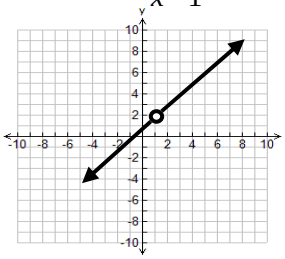
$f(x) = \dfrac{1}{x}$ (linear reciprocal)



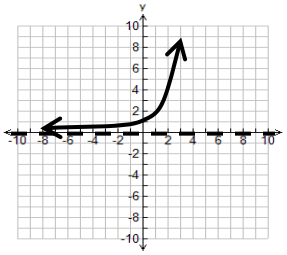
$g(x) = \sqrt{x}$ (square root)



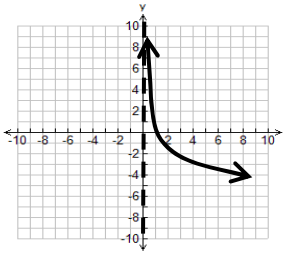
$m(x) = \dfrac{(x+1)(x-1)}{x-1}$ (hole)



Exponential function $f(x) = 2^x$

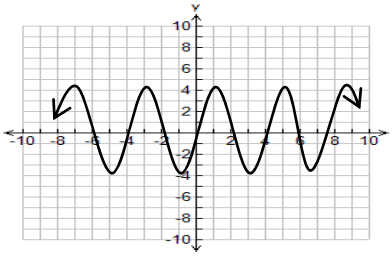


Logarithmic function $f(x) = \log_3 x$

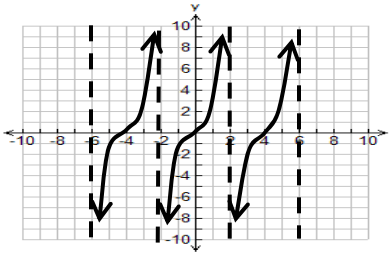


Trigonometric (Periodic) functions

$f(x) = \sin(\theta)$



$f(x) = \tan(\theta)$



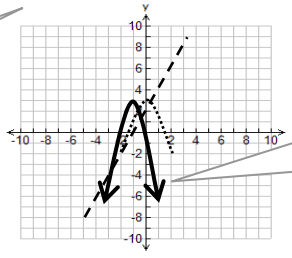
Other functions

If $f(x) = 2x + 2$

And $g(x) = -x^2 + 3$

Then $gof = g[f(x)] = -(2(x+1))^2 + 3$

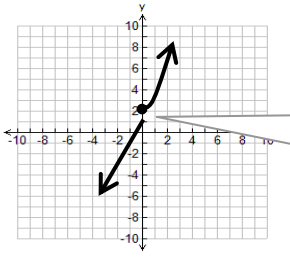
Composite function



Originals are dotted while composite is solid

$$m(x) = \begin{cases} 2x+1, x < 0 \\ x^2 + 2, x \geq 0 \end{cases}$$

Piecewise function



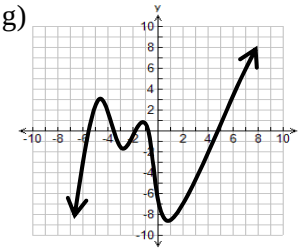
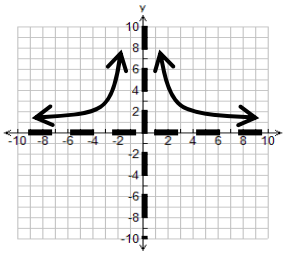
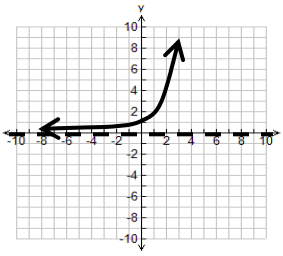
Notice discontinuity at x = 0

1.1B – Function Concepts Graphical Properties Practice Questions

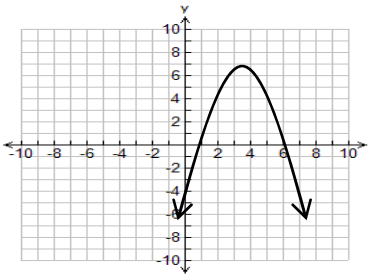
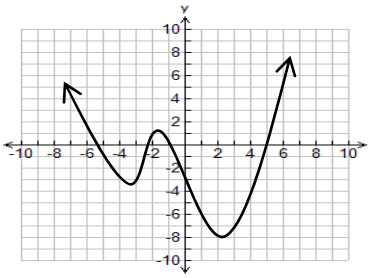
1. Identify the type of function (polynomial, rational, exponential, logarithmic, or trigonometric)

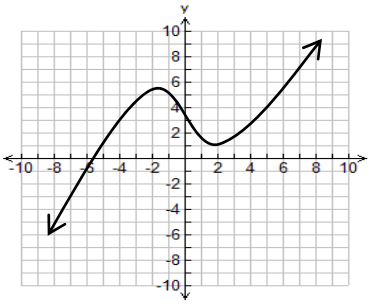
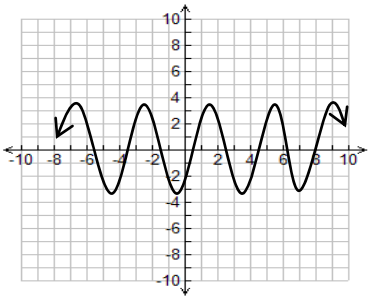
a) $f(x) = \sin(2x)$ b) $g(x) = -4x - 5$ c) $h(x) = 2^x - 1$

d) $f(x) = \dfrac{1}{2x-1}$ e) $y = \log_5 2x + 1$ f) $m(x) = 6x^3 - 2x^2 + 7$

g)  h)  i) 

2. State the zeros, increasing/decreasing intervals, local max/min and absolute maximum or minima for the following functions.

a)  b) 

c)  d) 

3. Set the following functions equal to zero and solve to find the zero(s) algebraically. State any problems you might encounter while trying to solve any of the resulting equations.

a) $f(x) = x^2 - 5x + 6$ b) $g(x) = x^3 - x^2 - 14x + 24$ c) $y = \dfrac{2x+3}{4x^2+4x-3} - 1$

Answers 1. a) trig **b)** poly **c)** expo **d)** rational **e)** log **f)** poly **g)** poly **h)** rational **i)** expo **2.** refer to table below
3. a) x=2,3 **b)** x=2,3,-4 **c)** x=1 (x=-1.5 is not a solution as this value is restricted from denominator)

Question	Zero(s)	Increasing	Decreasing	Local		Absolute	
				max	min	max	min
2a	1,6	x<3	-3<x & x>3	7	-	7	∞
2b	-5,-2,5	-3<x<-2 & 2<x	x<-2 & -2<x<2	1	-3,-8	∞	-8
2c	-5	x<-2 & 2<x	-2<x<2	6	1	∞	∞
2d	…-2,0,+2,…	-5+n<x<-3+n (where n is multiple of 2)	-3+n<x<-1+n (where n is multiple of 2)	3	-3	3	-3