1.2 – Transforming Functions

Transformations describe a set of processes that starts with an original function and then multiplies or adds to this function to create another function. Depending whether this is done inside or outside the function determines whether this effects the input (horizontal) or output (vertical) values. There are three graphical types of transformations.

Ex. $g(x) = \boldsymbol{f}(\text{inside})$ $g(x) = \text{outside } \boldsymbol{f}(\ \ ) + \text{outside}$

Translations involve adding (positive or negative real numbers) a value to the original function. This has the effect of moving the original function vertically (up or down) or horizontally (left or right)

g(x) is the transformed function of original f(x)

General form

Ex. $g(x) = f(x - \mathbf{h})$ $g(x) = (x - 3)^2$ — Moves horizontally by +3

Ex. $g(x) = f(x) + \mathbf{v}$ $g(x) = x^2 - 3$ — Moves vertically by -3

Specific example using quadratic

Stretching involves multiplying (dividing if multiplying by a number between zero and 1) the original function by some value. This has the effect of widening or compressing the original function. Since stretching vertically by 2 has same effect as compressing horizontally by ½ one really only needs to consider one (i.e. "a") stretch factor.

Ex. $g(x) = \mathbf{a}f(x)$ $g(x) = 9x^2$ a = 9

Ex. $g(x) = f(\mathbf{k}(x))$ — Stretch horizontally by 1/k $g(x) = (3x)^2$ k = 3

Same transformation in two different ways so really only need **a**

Reflection involves multiplying the original function by a negative. This has the effect of flipping the function vertically (about z-axis) or horizontally (about y-axis).

Ex. $g(x) = -fx$ $g(x) = -x^2$ — Flips vertically - outside

Ex. $g(x) = f(-x)$ $g(x) = (-x)^2$ — Flips horizontally - inside

Considered together we get a general form of how to apply transformations to a function

Ex. $g(x) = a\, f\,[k(x - h)] + v$ or just $g(x) = \boldsymbol{a}\, f\,(x - \boldsymbol{h}) + \boldsymbol{v}$

Stretch and reflection by a — Translate horizontally — Translate vertically

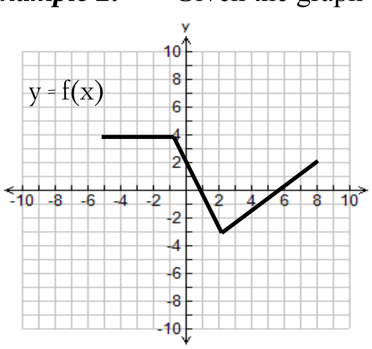
Example 1: Given $\boldsymbol{f(x) = x + 5}$ determine

Notice difference between b & c

Use brackets to avoid errors

a) h(x) = f(x) + 3	b) g(x) = 2f(x)	c) m(x) = f(2x)	d) r(x) = -3f(x – 2)
h(x) = (x + 5) + 3	g(x) = 2 (x + 5)	m(x) = (2x) + 5	r(x) = -3 [(x-2) – 3]
h(x) = x + 8	g(x) = 2x + 10	m(x) = 2x + 5	r(x) = -3x + 15

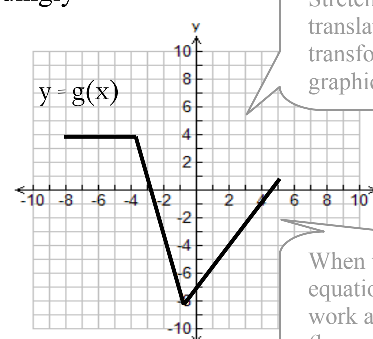
Example 2: Given the graph below transform the function accordingly



$g(x) = 2\, f\,(x + 3) - 2$

$(x, y) \rightarrow (x - 3, 2y - 2)$

Mapping notation shows algebraic steps to transform point.



Stretch/Flip then translate when transforming graphical function.

When transforming equation function work about vertex (key points)

1.2 – Transforming Functions Practice Questions

1. Given f(x) = x – 2, g(x) = (x + 3)^2, and $h(x) = -2\sqrt{x} + 1$ apply transformation to come up with a simplified version of new function.

a) m(x) = 2 f(x) b) m(x) = g(2x) c) y = 2 g(x) – 1 d) y = 2 h(x) + 3

e) y = f (2x – 1) + 3 f) $y = \dfrac{1}{g(x)}$ g) m(x) = *f o g* h) m(x) = 2 g [f(-x)]

2. Describe the transformation on the following functions according the examples show;

a) m(x) = 2 (-2(x + 3))^2 + 5

Translation:	Horizontal:	-3
	Vertical:	+5
Stretch:	Horizontal:	½
	Vertical:	2
Reflection:	Horizontal:	flip
	Vertical:	n/a

Horizontal by 1/k

b) g(x) = -3x + 2

Translation:	Horizontal:	n/a
	Vertical:	+2
Stretch:	Horizontal:	n/a
	Vertical:	3 (flip)

Combine stretch and reflection

c) f(x) = 3 sin (x – 60º) + 3

d) $h(x) = \dfrac{2}{x-3} + 5$

e) $g(x) = 2\log_5 3x + 4$

f) $y = -\dfrac{1}{2}x - 3$

g) $h(x) = \left(\dfrac{x}{4} - 1\right)^2 + 3$

h) $m(x) = 5 \cdot 2^{x-1} + 7$

i) $y = f[2(x-1)] + 3$

j) $y = -3f(x+4) - 5$

3. Express each of the following in mapping notation (as shown in question a) and then use graph of y = f(x) given to sketch the given transformations, each on a separate axis;

a) g(x) = 2 f(x) (x, y) → (x, 2y)

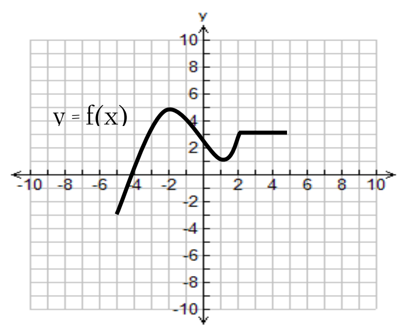
b) h(x) = 4 f(x) (x, y) → (

c) m(x) = f (x – 2) + 3

d) r(x) = -2 f(x + 1) – 2

e) p(x) = f $^{-1}$ (x)

f) y = f [½ (x – 1)]



Answers 1. a) m(x)=2x-4 **b)** m(x)=(2x+3)^2 **c)** y=2(x+3)^2-1 **d)** y=-4√x+5 **e)** y=2x **f)** y=1/(x+3)^2 **g)** m(x)=(x+3)^2-2 **h)** y=2[-1(x-1)]^2 **2.** refer to table below **3. b)** (x,4y) **c)** (x+2,y+3) **d)** (x-1,-2y-2) **e)** (y,x) **f)** (2x+1,y)

		a	b	c	d	e	f	g	h	i	J
function		quad	linear	trig	inverse	log	linear	quad	expo	n/a	n/a
translation	H	+3	-	+60°	+3	-	-	+4	+1	1	-4
	V	+5	+2	+3	+5	+4	-3	+3	+7	3	-5
stretch	H	-½	-	-	-	1/3	-	+4	-	½	-
	V	+2	-3	+3	+2	+2	-½	-	+5	-	-3