1.4 – Quadratic Functions

A ***quadratic function*** is the name give to a polynomial function of degree **2**. It is also known as the ***parabola***. The quadratic function is one of the most useful functions to describe (model) many natural phenomena including the gravitational effects on the height of objects.

Standard form: $y = ax^2 + bx + c$

a, b, c represent integer variables where b or c might be 0 and hence function might not contain these term

Factored form: $y = \mathbf{k}(x – a)(x - b)$

Useful form to quickly identify zeros.

Vertex form: $y = a(x – h)^2 + v$

Best form for graphing quickly. Vertex at (h, v) and stretch of +/- a.

Function notation: $f(x) = a(x – h)^2 + v$

Need to complete the square. Half the linear term squared (add and sub this value)

You could expand and then complete the square **or** try to work with equation in this form. It gives the zeros.

Example 1: Sketch the following:

a) $f(x) = -2(x +1)^2 - 3$

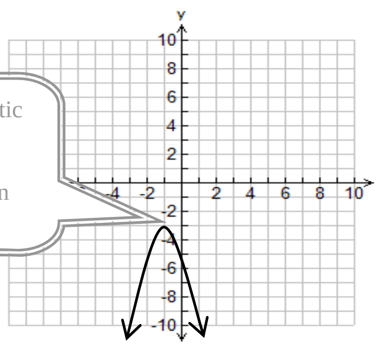
Vertex at (-1, -3)
Stretch of -2 (1 over, and -2)

b) $f(x) = x^2 +6x + 8$

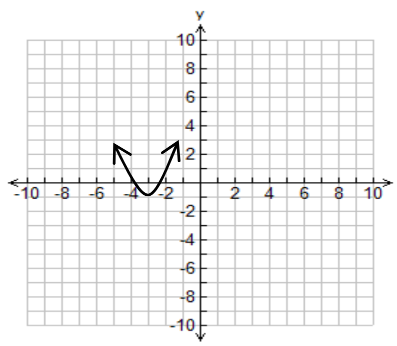
$$f(x) = x^2 + 6x + 9 - 9 + 8$$
$$f(x) = (x+3)^2 - 1$$

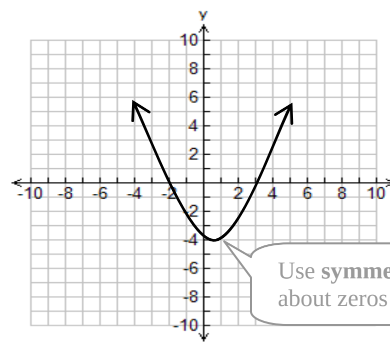
c) $f(x) = 2(x +2)(x – 3)$

This works fine for sketching but will not give accurate stretch transformation.



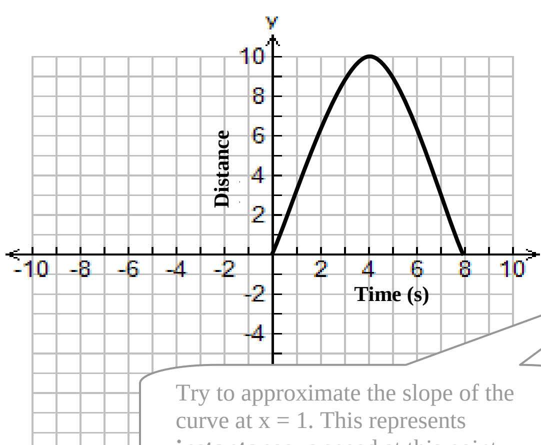
Translate quadratic vertex (0,0)
Stretch one patterned point on either side





Use **symmetry** about zeros

Example 2: Given the graph below calculate;



Try to approximate the slope of the curve at x = 1. This represents **instantaneous** speed at this point.

This is **average** slope (i.e. speed) between 0 and 2 seconds

a) Slope from 0 to 2 seconds m = 3

b) Slope from 0 to 4 seconds m = 2.5

Notice speed is slowing

c) Slope at 1 seconds m = 3.5
d) Slope at 2 seconds m = 2
e) Slope at 3 seconds m = 1
f) Slope at 4 seconds m = 0
g) Slope at 6 seconds m = -2

1.4 – Quadratic Functions Practice Questions

1. Graph the following using transformations.

   a) $f(x)=(x-1)^2+2$ b) $m(x)=2(x-1)^2$ c) $h(x)=(\frac{1}{2}x)^2-1$

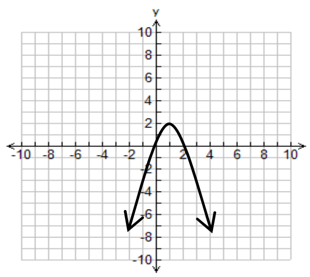
   d) $f(x)=\frac{1}{2}x^2-1$ e) $y=-20(x+200)^2-30$ f) $g(x)=(-\frac{1}{2}x+1)^2-2$

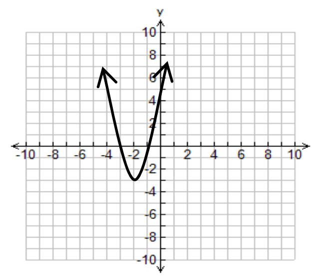
   g) $y=-2(-2(x-2))^2-2$ h) $f(x)=-2x^2+4x$ i) $h(x)=x^2-60x+800$

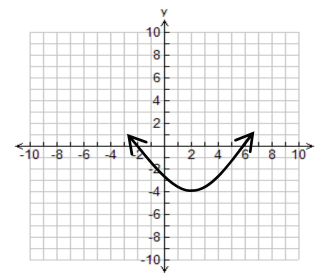
2. Graph the following using zeros.

   a) f(x) = (x – 2)(x+ 1) b) g(x) = -(x – 3)(x + 4) c) y = x(x + 30)
   d) m(x) = (2x – 1)(x – 3) e) $y = -2x^2 + 4x$ f) $y = x^2 - x - 6$

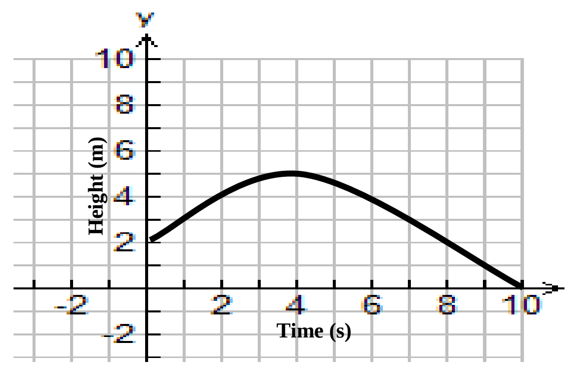
3. Given the following graphs determine the quadratic function that best models the curve.

   a) 

   b) 

   c) 

4. The diagram below describes the flight of a football thrown between a quarterback and the intended receiver. Determine;



   a) the maximum height the football reaches
   b) the initial height from which football was thrown
   c) at what time the ball hits the ground
   d) the ***average rate*** at which the balls' height changes from; i) 0 to 1s ii) 0 to 3s iii) 0 to 4s iv) 2 to 6s
   e) the ***instantaneous rate*** at which the balls' height changes at; i) 0s ii) 3s iii) 4s iv) 10s

5. The height of an object dropped from an initial height, h_o, above the ground is given by the function $\boldsymbol{h(t) = -5t^2 + h_o}$, where *h(t)* is measured in meters and *t* in seconds.

   a) Calculate the height of an object, after 6 seconds, if dropped from 200m.
   b) If an object takes 2.5s to hit the ground, calculate its initial height.
   c) How long will it take an object dropped form 70m to hit the ground.
   d) What is the functional interpretation of question c?

Answers 3. a) $f(x)=-(x-1)^2+2$ **b)** $y=2(x+2)^2-3$ **c)** $g(x)=[½(x-2)]^2-4$ **4. a)** 5m **b)** 2m **c)** 10s **d) i)** 1m/s **ii)** 0.9m/s **iii)** 0.8m/s **iv)** 0m/s **e) i)** 1m/s **ii)** 0.5m/s **iii)** 0m/s **iv)** -1m/s **5. a)** 20m **b)** 31.25m **c)** 3.7s

d) $t(h)=\sqrt{\frac{h_o-h}{5}}$ inverse function gives time as a function of initial height and height off ground.

1.4 - Sketching Practice Sheet