

1.5A – Polynomial Functions Introduction

Overall degree of n

Linear term

Constant term

Standard form: $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x^1 + a_0$

Factored form: $f(x) = k(x - a)(x - b)(x - c)(x + d)$

k is a constant that is used to represent a family of functions that all have the same zeros

To sketch the graph of a polynomial function one will need to identify its' key features;

- general shape - from overall degree of function
- direction of opening - sign in front of highest degree term
- the zeros (where curve crosses x-axis) - determined from factored form

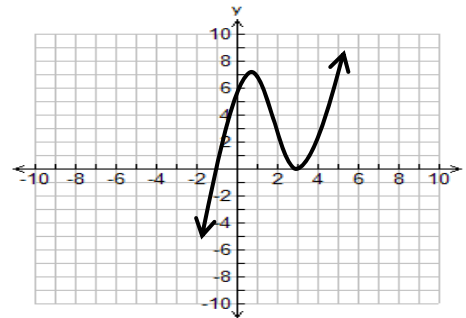
Make notes on the following chart to outline some general observations when graphing the following polynomial functions on a graphing calculator

Type of function	Degree	Example	Factored	# of Zeros	Other notes
linear	1	$f(x) = 2x + 4$	$f(x) = 2(x + 2)$	0 or 1	1 positive section
quadratic	2	$g(x) = -x^2 - 5x - 6$	$g(x) = -(x + 2)(x + 3)$	0, 1 or 2	Opens down
cubic	3	$h(x) = x^3 - 2x^2$	$h(x) = x(x - 2)$	1, 2, or 3	Touch point at $x = 0$
quartic	4	$m(x) = x^4 - 4x^3 - 12x^2$	$m(x) = x^2(x - 6)(x + 2)$	0 to 4	4 sections
fifth	5	$f(x) = ?$	$f(x) = (x - 2)^2(x + 3)(x - 1)(x + 2)$	1 to 5	Touch point at $x = 2$
sixth	6	$g(x)$	$g(x) = x^2(x - 3)^2(x - 1)(x + 2)$	0 to 6	6 possible sections

Example 1: Sketch the following polynomial functions;

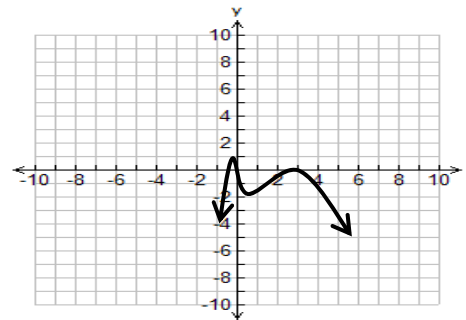
a) $g(x) = (x - 3)^2(x + 1)$

opens: does not apply to odd degree functions
 degree: 3rd (so 3 possible sections starting with positive)
 $x = -1$ (crosses x-axis)
 $x = 3$ (touch point on x-axis)



b) $f(x) = -2x(3x + 1)(x - 3)^2$

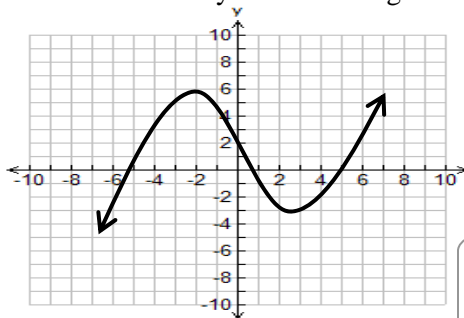
open: down
 degree: 4th (so 4 possible sections)
 zeros: crosses at $x = -1/3$, $x = 0$
 touches at $x = +3$



To find zeros set each factored bracket to zero.
 So $3x + 1 = 0$
 $3x = -1$
 $x = -1/3$

Example 2: Determine a family of functions given the following graph

a)



Zeros are at $x = -5, +1, +5$

$g(x) = k(x + 5)(x - 1)(x - 5)$

How could you use max at $(-2, 6)$ to find value of k ?

Need to use opposite sign when write in factored form. Why?

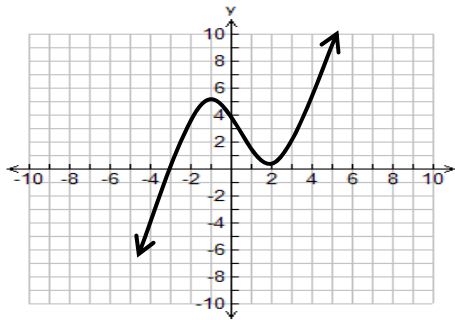
1.5A – Polynomial Functions Introduction Practice Questions

1. Sketch the following functions. Check using graphing calculator if available.

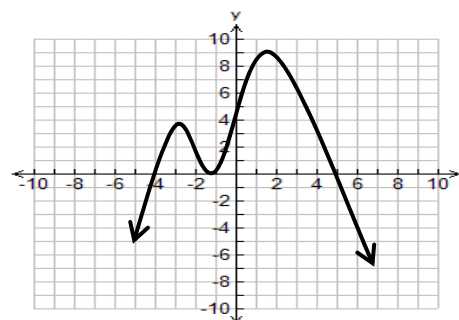
- a) Positive 3rd degree function that has zeros at -2, 0 and +2.
- b) $f(x) = x(x - 2)(x + 4)^2$
- c) Negative 2nd degree function that has zeros at -2 and +5
- d) $f(x) = x^4$
- e) $f(x) = x^4 + 3$
- f) $f(x) = (x + 3)^4$
- g) $f(x) = x^4 + 3x^3$
- h) $f(x) = x^2(x + 2)(x - 1)$
- i) $f(x) = -(x + 3)^2(x - 1)^2$
- j) $g(x) = -x^3$
- k) $g(x) = (x + 2)^2(x - 1)^2$
- l) $y = x^2 - x - 12$
- m) $h(x) = -x^3 - 9x$
- n) $m(x) = x^4 - 3x^2 + 2$

2. Determine the family of the functions, in form $f(x) = k(x - a)(x - b)(x - c)(x + d)$, given the information below;

- a) 3rd degree function has zeros at -2, +2 and +3
- b) 4th degree function touches the x-axis at -4 crosses x-axis at 0 and +2.
- c) 5th degree function that touches x-axis at -2, -3 and crosses at +5
- d) 2nd degree function that touches x-axis at +3
- e) 3rd degree function with zeros at $-1/2$, $+1/4$ and -3
- f) Use graph below



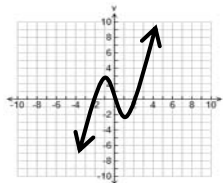
g) use graph below



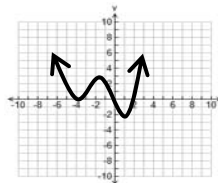
3. Sketch the function $f(x) = -x(x - 4)(x + 4)$ and then determine the values of any relative maximums or minimums.

4. Sketch the function $g(x) = x^2(x - 2)(x + 2)$ and then determine the values of any relative maximums or minimums.

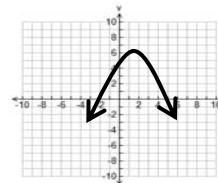
Answers 1. a)



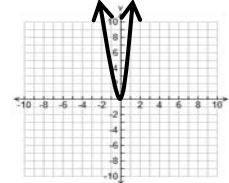
b)



c)



d)



e) check other graphs using graphing calculator

2. a) $f(x) = k(x - 2)(x + 2)(x - 3)$ b) $f(x) = kx(x + 4)^2(x - 2)$ c) $f(x) = k(x + 2)^2(x + 3)^2(x - 5)$ d) $f(x) = k(x - 3)^2$

e) $f(x) = k(2x + 1)(4x - 1)(x + 3)$ f) $f(x) = k(x + 3)(x - 2)^2$ g) $f(x) = k(x + 4)(x + 1)^2(x - 5)$

3. $f(2) = 24$ is a local max, $f(-2) = -24$ is a local min 4. $g(0) = 0$ is local max, $g(-1) = -3$ and $g(1) = -3$ are local mins, but if you use the calculator to trace you can more accurately see $g(-1.4) = -3.9$ and $g(1.4) = 3.9$

1.5A - Sketching Practice Sheet

