1.6 – Polynomial Functions from Data

A difference table can be used to analyze whether a given set of data points fits into a polynomial function model. A graphing calculator can perform various regressions to determine the strength of this model.

Example 1: Place the following data into a difference table and come up with an algebraic model for these data.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
<th>Δf(x)</th>
<th>Δ²f(x)</th>
<th>Δ³f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>11</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>15</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>19</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>49</td>
<td>23</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>72</td>
<td>27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>99</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

So \( f(x) = ax^2 + bx + c \) and then know \( f(1) = 4 \) so \( a(1)^2 + b(1) + c = 4 \) ①
\( f(2) = 15 \) \( a(2)^2 + b(2) + c = 15 \) ②
\( f(3) = 30 \) \( a(3)^2 + b(3) + c = 30 \) ③

Solve using elimination ③ - ① \( 3a + b = 11 \) ④
③ - ② \( 5a + b = 15 \) ⑤
⑤ - ④ \( 2a = 4 \) or \( a = 2 \)

Sub \( a = 2 \) back into ④ yields \( 5(2) + b = 15 \) or \( b = 5 \)
Sub \( a = 2 \) and \( b = 5 \) into ① yields \( 2 + 5 + c = 4 \) or \( c = -3 \)

Therefore functions is; \( f(x) = 2x^2 + 5x - 3 \)

Example 2: The following data tracked the population in a small town over a 6 year period. Place the data into a difference table as well as a graphing calculator to come up with an algebraic model for this data. Then use the model to estimate what the population was in 1979 and 1990.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Populations (y)</td>
<td>4031</td>
<td>4008</td>
<td>3937</td>
<td>3824</td>
<td>3675</td>
<td>3496</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
<th>Δf(x)</th>
<th>Δ²f(x)</th>
<th>Δ³f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>4031</td>
<td>-23</td>
<td>-48</td>
<td>6</td>
</tr>
<tr>
<td>1982</td>
<td>4008</td>
<td>-71</td>
<td>-42</td>
<td>6</td>
</tr>
<tr>
<td>1983</td>
<td>3937</td>
<td>-113</td>
<td>-36</td>
<td>6</td>
</tr>
<tr>
<td>1984</td>
<td>3824</td>
<td>-149</td>
<td>-30</td>
<td>6</td>
</tr>
<tr>
<td>1985</td>
<td>3675</td>
<td>-179</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1986</td>
<td>3496</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A constant third difference points to a 3rd degree relation.

Data says that when \( x=1 \) \( y=4 \) or \( f(1)=4 \)
Difference table show a 3\textsuperscript{rd} degree relation which is confirmed by a regression value of 1 on the graphing calculator.

So we know it will have the general form: 

\[ p(y) = ay^3 + by^2 + cy + d \]

The graphing calculator gives the following:

\begin{align*}
a &= 1 \\
b &= -5970 \\
c &= 11880060 \\
d &= -7880118800
\end{align*}

Large values would make algebraic calculation more difficult. One could adjust years to zero (i.e. 1981 \rightarrow 0 and 1982 \rightarrow 1, etc)

\[ R^2 = 1 \]

So our equation is:

\[ p(y) = y^3 - 5970y^2 + 11880060y - 7880118800 \]

And so 1979 

\[ p(1979) = (1979)^3 - 5970(1979)^2 + 11880060(1979) - 7880118800 = 3909 \]

\[ p(1990) = 2600 \]

1.6 – Polynomial Functions from Data Practice Questions

1. Determine, by algebraic methods, a polynomial function that models the data below;

\begin{enumerate}
  \item a) \((1,0), (2,-2), (3,-2), (4,0), (5,4), (6,10)\)
  \item b) \((1,-1), (2,2), (3,5), (4,8), (5,11), (6,14)\)
  \item c) \((1,-4), (2,0), (3,30), (4,98), (5,216), (6,396)\)
  \item d) \((1,-9), (2,-10), (3,-7), (4,0), (5,11), (6,26)\)
\end{enumerate}

2. Use a graphing calculator to determine the best polynomial function that models the data. Note that the data does not necessarily have a perfect (regression value of 1) fit to one polynomial model

\begin{enumerate}
  \item a) \((1,1), (2,-3), (3,5), (4,37), (5,105), (6,221)\)
  \item b) \((0,4), (1,16), (2,30), (3,60), (4,120), (5,250)\)
  \item c) \((1,-9), (2,-31), (3,-31), (4,51), (5,299), (6,821)\)
\end{enumerate}

**Answers**

1. \(a) f(x) = x^2 - 5x + 4 \)  \(b) g(x) = 3x - 4 \)  \(c) h(x) = 2x^3 + x^2 - 13x + 6 \)  \(d) m(x) = 2x^2 - 7x - 4 \)

2. \(a) y=2x^2-6x^2+5 \)  \(R=1 \)  \(b) y=3.5x^3-12.7x^2+24.3x+3.2 \)  \(R=0.99 \)  \(c) y=x^4-14x^2+5x-1 \)  \(R=1 \)

1.6 – polynomial functions from data