1.6 – Polynomial Functions from Data

A difference table can be used to analyze whether a given set of data points fits into a polynomial function model. A graphing calculator can perform various regressions to determine the strength of this model.

Example 1: Place the following data into a difference table and come up with an algebraic model for these data.

x	1	5	3	2	6	4
f(x)	4	72	30	15	99	49

> 1^{st} difference is calculated by subtracting two consecutive output values (i.e. $\Delta y = y_2 - y_1$) **or** lower minus upper value

> Data needs to be organized sequentially. To be useful input (x) values also have to have a constant difference

x	f(x)	Δf(x)	Δ^2f(x)	Δ^3f(x)
1	4			
2	15	11	4	
3	30	15	4	
4	49	19	4	
5	72	23	4	
6	99	27		

> A constant second difference points to a 2^{nd} degree relation.

> Data says that when x=1 y=4 or f(1)=4

So ***f(x) = ax² + bx + c*** and then know

$f(1) = 4$ so $a(1)^2 + b(1) + c = 4$ ①
$f(2) = 15$ $a(2)^2 + b(2) + c = 15$ ②
$f(3) = 30$ $a(3)^2 + b(3) + c = 30$ ③

> Label equations so other can follow your work

Solve using elimination

② - ① $3a + b = 11$ ④
③ - ② $5a + b = 15$ ⑤

⑤ - ④ $2a = 4$ or $a = 2$

Sub a = 2 back into ⑤ yields $5(2) + b = 15$ or $b = 5$
Sub a = 2 and b = 5 into ① yields $2 + 5 + c = 4$ or $c = -3$

Therefore functions is; $f(x) = 2x^2 + 5x - 3$

Example 2: The following data tracked the population in a small town over a 6 year period. Place the data into a difference table as well as a graphing calculator to come up with an algebraic model for this data. Then use the model to estimate what the population was in 1979 and 1990.

Year (x)	1981	1982	1983	1984	1985	1986
Populations (y)	4031	4008	3937	3824	3675	3496

x	f(x)	Δf(x)	Δ^2f(x)	Δ^3f(x)
1981	4031			
1982	4008	-23	-48	6
1983	3937	-71	-42	6
1984	3824	-113	-36	6
1985	3675	-149	-30	
1986	3496	-179		

> A constant third difference points to a 3^{rd} degree relation.

Difference table show a 3rd degree relation which is confirmed by a regression value of 1 on the graphing calculator

So we know it will have the general form: $p(y) = ay^3 + by^2 + cy + d$

The graphing calculator gives the following:

$a = 1$
$b = -5970$
$c = 11880060$
$d = -7880118800$

$R^2 = 1$

R value indicates a strong postive correlation

Large values would make algebraic calculation more difficult. One could adjust years to zero (i.e. 1981→0 and 1982→ 1, etc)

So our equation is: $p(y) = y^3 - 5970y^2 + 11880060y - 7880118800$

Population as a function of year

And so 1979 $p(1979) = (1979)^3 - 5970(1979)^2 + 11880060(1979) - 7880118800$
$= 3909$

$p(1990) = 2600$

1.6 – Polynomial Functions from Data Practice Questions

1. Determine, by algebraic methods, a polynomial function that models the data below;

   a) (1,0), (2,-2), (3,-2), (4,0), (5,4), (6,10)
   b) (1,-1), (2,2), (3,5), (4,8), (5,11), (6,14)
   c) (1,-4), (2,0), (3,30), (4,98), (5,216), (6,396)
   d) (1,-9), (2,-10), (3,-7), (4,0), (5,11), (6,26)

2. Use a graphing calculator to determine the best polynomial function that models the data. Note that the data does not necessarily have a perfect (regression value of 1) fit to one polynomial model

   a) (1,1), (2,-3), (3, 5), (4,37), (5,105), (6,221)
   b) (0,4), (1,16), (2,30), (3,60), (4,120), (5,250)
   c) (1,-9), (2,-31), (3,-31), (4,51), (5,299), (6,821)

Answers 1. a) $f(x)= x^2 - 5x + 4$ **b)** $g(x) = 3x - 4$ **c)** $h(x) = 2x^3 + x^2 - 13x + 6$ **d)** $m(x) = 2x^2 - 7x - 4$
2. a) $y=2x^3-6x^2+5$ R=1 **b)** $y=3.5x^3-12.7x^2+24.3x+3.2$ R=0.99 **c)** $y=x^4-14x^2+5x-1$ R=1