## 1.7 – Reviewing Polynomial Functions

***Function*** – a special relation in which each element of the domain (x-value) corresponds to only one element of the range (y-value). A function will pass the Vertical Line Test (VLT)

***Evaluating*** a function is best done using function notation. This tells one the value of the function (y-value) for a given input (x-value).

Ex. Given $f(x) = 3x + 1$ then evaluating at $x = 5$

$$f(5) = 3(5) + 1 = 16$$

***Composite functions*** are made when two or more functions are combined. Some specific notation helps one keep track of these combining steps.

Ex. Given two functions *f(x)* and *g(x)*

$f \circ g$ means $f[g(x)]$
$g \circ f$ means $g[f(x)]$

***Inverting a function*** implies undoing what was done.

Ex. If $m(x) = 2x - 3$ then its inverse $m^{-1}(x) = (x + 3) \div 2$

***Zeros*** – the domain (x-value) location(s) where the function (y-value) equals zero.

***Increasing and Decreasing*** intervals talk about whether the function increases or decreases it value over a given domain.

***Maximum or Minimum*** values of a function occur locally (relatively) at peaks (max) or valleys (min). One is interested in stating the function output (y-value) at the given domain input. The absolute max/min considers the largest or smallest value that the function ever reaches.

***Stretching, reflecting and translating*** are transformation processes that take any function and transform it into a new function

Ex. $g(x) = a\,f\,[k(x - h)] + v$ or just $g(x) = \mathbf{a}\,f\,(x - \mathbf{h}) + \mathbf{v}$

Translate vertically
Stretch and reflection by a
Translate horizontally

***Linear function***:

| | |
|---|---|
| Slope intercept form: | $y = mx + b$ |
| In function notation form: | $f(x) = ax + b$ |
| Standard form: | $Ax + By + C = 0$ |

***Rate of change*** tells one how the function increases or decreases with respect to x. Basically, it is just a real life way to think about slope.

Ex. $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$ better known as $slope = \dfrac{rise}{run}$

***Quadratic function:***

| | | |
|---|---|---|
| | Standard form: | $y = ax^2 + bx + c$ |
| | Factored form: | $y = \mathbf{k}(x - a)(x - b)$ |
| | Vertex form: | $y = a(x - h)^2 + v$ |
| | Function notation: | $f(x) = a(x - h)^2 + v$ |

***Polynomial function:***

| | | |
|---|---|---|
| | Standard form: | $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1x^1 + a_0$ |
| | Factored form: | $f(x) = \mathbf{k}(x - a)(x - b)(x - c)(x + d)$ |

***To sketch*** a polynomial function one needs to identify key features, including;

a) general shape - from overall degree of function
b) direction of opening - sign in front of highest degreed term
c) the zeros (where curve touches or crosses x-axis) - determined form factored form
d) use symmetry to sketch between the zeros
e) Sometimes have to skew graph between zeros triple or higher exponent

***A difference table*** can be used to analyze whether a given set of data points fits into a polynomial function model. A graphing calculator can perform various regressions to determine the strength of this model.

**1.7 – Reviewing Polynomial Functions Practice Questions**

1. Given the functions listed below, evaluate the following;

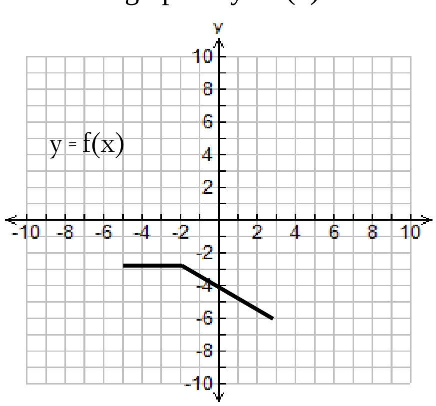
$h(x) = x + 2$ $\qquad g(x) = (x-1)^2 + 2$ $\qquad m(x) = \dfrac{2x-1}{3}$ $\qquad f(x) = 3\sqrt{2-x} + 1$

| | | | |
|---|---|---|---|
| a) h(-5) | b) g(2) | c) m(5) | d) f(-7) |
| e) *f o g (1)* | f) h o g (3) | g) *g o f (2)* | h) *f o g (-3)* |
| i) *h o g o m (5)* | j) $m^{-1}(7)$ | k) $g^{-1}$[h(m(2))] | l) g{h[f(m(2))]} |

2. Describe the transformation on the following functions;

a) $m(x) = 4\left(-\tfrac{1}{4}(x + 1)\right)^2 + 2$ $\qquad$ b) $g(x) = \left(-\dfrac{1}{3}x\right)^3 - 2$ $\qquad$ c) f(x) = 4 sin (3x – 180º) + 2

3. Given the graph of y = f(x) sketch each of the following on a separate axis;




a) g(x) = 2 f(x)

b) h(x) = 4 f(x)

c) m(x) = f (x – 2) + 3

d) r(x) = -2 f(x + 1) – 2

e) $p(x) = f^{-1}(x)$

4. Graph the following using the method of your choice.

a) $f(x) = 4x - 50$ b) $g(x) = \frac{2}{3}x + 1$ c) $h(x) = (\frac{1}{2}x)^2 - 1$

d) $f(x) = \frac{1}{2}x^2 - 1$ e) $g(x) = (-\frac{1}{2}x + 1)^2 - 2$ f) $f(x) = (x - 2)(x + 1)$

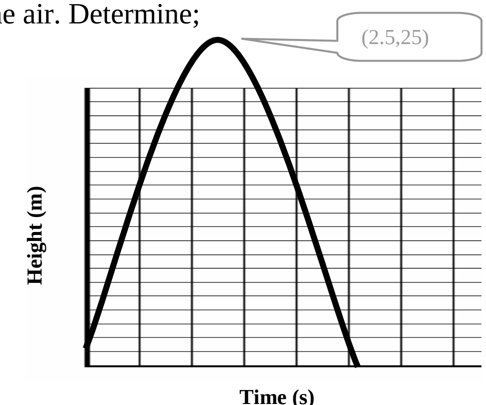
g) $h(x) = x^2 - 60x + 800$ h) $y = -2x^2 + 4x$ i) $f(x) = (x - 3)(x + 4)^3(x - 1)$

j) $h(x) = (2x - 1)^3(x - 5)$ k) $g(x) = 3x^4 - 3x^2$ l) $m(x) = (0.5(x - 3))^3 + 1$

5. Determine the exact equation of the function given the information below;

a) $f(x) = k(x - 1)(x + 2)$ and goes through point (2, -8)
b) $g(x) = kx^2(x - 2)$ and $g(-1) = -6$.
c) 3rd degree function has zeros at -2, +2 and +3 and passes through point (0, 12)
d) 4th degree function touches the x-axis at -4 crosses x-axis at 0 and +2. (-1, 54) is a point on function.

6. The diagram below describes the height of a ball thrown into the air. Determine;




a) the maximum height the ball reaches
b) the initial height from which ball was thrown
c) Come up with an equation to describe balls height
d) at what time the ball hits the ground
e) the ***average rate*** at which the balls' height changes from; i) 0 to 1s ii) 0 to 4s
f) the ***instantaneous rate*** at which the balls' height changes at; i) 1s ii) 4s

7. The height of an object dropped from an initial height, $h_o$, above the ground is given by the function $\boldsymbol{h(t) = -5t^2 + h_o}$, where $h(t)$ is measured in meters and $t$ in seconds.

a) Calculate the height of an object, after 3 seconds, if dropped from 150m.
b) If an object takes 5s to hit the ground, calculate its initial height.
c) How long will it take an object dropped form 500m to hit the ground?
d) What is the functional interpretation of question c?

8. When the contractions of a pregnant woman reach 5 minute intervals it is time to go to the hospital to prepare for delivery. At 8:00am a woman started to have contractions and made the notes as outlined in the table below. Organize this data to come up with the best polynomial function model which you can then use to determine at what time the woman should leave for the hospital.

| Time of day | 08:00 | 09:00 | 09:50 | 12:00 | 14:10 |
|---|---|---|---|---|---|
| Contraction Intervals (minutes) | - | 60 | 50 | 30 | 20 |

**Answers 1. a)** -3 **b)** 3 **c)** 3 **d)** 10 **e)** 1 **f)** 8 **g)** 2 **h)** no real solution **i)** 8 **j)** 11 **k)** 2 **l)** 27 **5. a)** f(x)=-2(x-1)(x+2) **b)** g(x)=-2xx(x-2) **c)** y=(x+2)(x-2)(x-3) **d)** $y=2x(x+4)^2(x-2)$ **6. a)** 25m **b)** 1m **c)** about 5s **d)** $h(t)=-4(t-25)^2+25$ **e)** 11m/s, 2.75m/s **f)** 11m/s, -11m/s **7. a)** 105m **b)** 125m **c)** 10s **d)** inverse function gives time as function of height **8.** $I(t)=0.000131t^2-0,2065t+71.6$ or $I(t)=0.000131(t-790)^2-9.4$ gives intervals as a function of time, t minutes after 8:00am **b)** 458 minutes after 8:00am or 15:38 or 3:38pm

## 1.7 - Sketching Practice Sheet