2.1 – Factoring Concepts

Common factoring:

a) $9x^3+6x^5+12x^2$

$= 3x^2(3x + 2x^3 + 4)$

b) $8xy^2 + 4x^2y - 6xy^5$

$= 2xy(4y + 2x - 3y^4)$

Grouping:

a) $2x^2 - 6x + 3xy - 9y$

$= 2x(x - 3) + 3y(x - 3)$

$= (2x + 3y)(x - 3)$

b) $3x^2 - 6x - x + 2$

$= 3x(x - 2) - 1(x - 2)$

$= (3x - 1)(x - 2)$

Careful with -1 factor

Simple Trinomial:

a) $x^2 - 2x - 8$

$= (x + 2)(x - 4)$

Two numbers that add to -2 and multiple to +8. Thinking of multiplier factors (numbers) first is easier.

b) $3x + 28 - x^2$

Rewrite in descending order

$= -x^2 + 3x + 28$

$= -1[x^2 - 3x - 28]$

Factor out -1

$= -(x - 7)(x + 4)$

c) $x^2 + 2xy - 8y^2$

$= (x - 2y)(x + 4y)$

Still follows basic pattern. Expand brackets if you want to verify.

Trinomial Decomposition:

a) $5x^2 - 12x + 4$

$= 5x^2 - 10x - 2x + 4$

$= 5x(x - 2) - 2(x - 2)$

$= (5x - 2)(x - 2)$

This time think of two numbers that add to -12 and multiple to +20 (5 x 4) Then use grouping technique.

b) $9a^2 - 30ab + 25b^2$

$= 9a^2 - 15ab - 15ab + 25b^2$

$= 3a(3a - 5b) - 5b(3a - 5b)$

$= (3a - 5b)^2$

You might recognize this as a perfect trinomial square and know the quick way to factor?

Difference of Squares:

a) $m^2 - 16$

$= (m - 4)(m + 4)$

b) $9x^2 - 5y^2$

$= (9x - \sqrt{5}y)(9x + \sqrt{5}y)$

Can still factor just leave as radical.

Sum or Difference of Cubes:

a) $x^3 - y^3$

$= (x - y)(x^2 + xy + y^2)$

Two new patterns for you.

b) $8x^3 + 27y^3$

$= (2x + 3y)(4x^2 - 6xy + 9y^2)$

Other Cases:

a) $4a^2 + 4ab + b^2 - 1$

$= (2a + b)^2 - 1$

$= (2a + b - 1)(2a + b + 1)$

Sometimes it helps to look for groupings?

b) $x^2 - 2 + \frac{1}{x^2}$

$= \frac{1}{x^2}(x^4 - 2x^2 + 1)$

$= \frac{1}{x^2}(x^2 - 1)^2$

$= \frac{1}{x^2}(x - 1)^2(x + 1)^2$

Can factor out fraction if want or ?

2.1 – Factoring Concepts Practice Questions

1. Factor fully. General order you look to factor in should be; common factor, grouping, trinomial, decomposition, special cases.

a) $3x – 6$	b) $15x + 10y + 25$	c) $4x – 4y + 8$	d) $12x^2 – 6x + 9$
e) $x^2 – 6x - 7$	f) $x^2 + 7x + 10$	g) $x^2 – 5x - 36$	h) $x^2 –5xy + 6y^2$
i) $x^2+ 4x + 4$	j) $x^2 – 6x + 9$	k) $x^2 – 2x + 1$	l) $y^2 +8y + 16$
m) $x^2 – 25$	n) $x^2 - 9$	o) $x^2 – 49$	p) $4x^2 - 25$
q) $3x^2 – 11x – 20$	r) $2y^2 + 7y + 3$	s) $5x^2 + x – 18$	t) $3x^2 + 16xy + 5y^2$

2. Factor fully. General order you look to factor in should be; common factor, grouping, trinomial, decomposition, special cases.

a) $49x^2 - 64$	b) $49x^2 – (x – y)^2$	c) $y^3 + y^2 + y + 1$
d) $27a^2 – 48$	e) $9x^4 + 12x^2 + 4$	f) $p^4 + 2p^2q^2 + q^4$
g) $\sin^2x – \sin x – 6$	h) $6e^{2x}+ 19e^x + 15$	i) $8x^2(x+1) + 2x(x + 1) – 3(x + 1)$
j) $x^3 – 27$	k) $8x^3 + 125$	l) $3x^3 – 24y^3$

Answers 1.a) $3(x–2)$ **b)** $5(3x+2y+5)$ **c)** $4(x–y+2)$ **d)** $3(4x^2–2x+3)$ **e)** $(x-7)(x+1)$ **f)** $(x+5)(x+2)$ **g)** $(x-9)(x+4)$ **h)** $(x-2y)(x-3y)$ **i)** $(x+2)^2$ **j)** $(x-3)^2$ **k)** $(x-1)^2$ **l)** $(y+4)^2$ **m)** $(x-5)(x+5)$ **n)** $(x-3)(x+3)$ **o)** $(x-7)(x+7)$ **p)** $(2x-5)(2x+5)$ **q)** $(3x+4)(x-5)$ **r)** $(2y+1)(y+3)$ **s)** $(5x-9)(x+2)$ **t)** $(3x+y)(x+5y)$ **2. a)** $(7x-8)(7x+8)$ **b)** $(6x+y)(8x-y)$ **c)** $(y^2+1)(y+1)$ **d)** $3(3a–4)(4a+4)$ **e)** $(3x^2+2)^2$ **f)** $(p^2+q^2)^2$ **g)** $(\sin x-3)(\sin x+2)$ **h)** $(2e^x-3)(3e^x-5)$ **i)** $(2x-1)(4x+3)(x+1)$ **j)** $(x-3)(x^2+3x+9)$ **k)** $(2x+5)(4x^2-10x+25)$ **l)** $3(x-2y)(x^2+2xy+4y^2)$