2.3 – Multiplying Rational Expressions

The same rules apply to multiplying rational expressions as to when multiply any rational numbers (i.e. fractions).

a) multiply across (numerator with numerator and denominator with denominator)
b) Reduce to lowest terms by dividing out any common factors
c) State any restrictions. You will need to state restriction on both top and bottom for any divisor term as it gets flipped during the operation.

Example 1: Simplify the following expression. State all restrictions.

a)

$$\frac{3x^3}{2y^2}\times\frac{10y^3}{9x^2}$$

$$=\frac{30x^3y^3}{18x^2y^2}$$

$$=\frac{5xy}{3},\quad x,y\neq 0$$

To avoid large numbers you can cancel out before multiplying

b)

$$\frac{2ab}{5c}\div\frac{14a^2b^2}{15c^2}$$

$$=\frac{2ab}{5c}\times\frac{15c^2}{14a^2b^2}$$

$$=\frac{1c}{7ab},\quad a,b,c\neq 0$$

c)

$$\frac{x^2+x-6}{x^2+2x-15}\times\frac{x-3}{x-2}$$

$$=\frac{(x+3)(x-2)}{(x+5)(x-3)}\times\frac{(x-3)}{(x-2)}$$

$$=\frac{x+3}{x+5},\quad x\neq -5,2,3$$

Brackets indicate that entire package must be considered together.

d)

$$\frac{x^2-x-20}{x^2-6x}\div\frac{x^2+9x+20}{x^2-12x+36}$$

$$=\frac{(x-5)(x+4)}{(x)(x-6)}\times\frac{x^2-12x+36}{x^2+9x+20}$$

$$=\frac{(x-5)(x+4)}{(x)(x-6)}\times\frac{(x-6)(x-6)}{(x+4)(x+5)}$$

$$=\frac{(x-5)(x-6)}{x(x+5)},\quad x\neq -5-4,0,6$$

Invert and multiply

Restrictions are easiest to get from the factored line

formally stating restrictions (partial example)

$$x^2+9x^2+20\neq 0$$

$$(x+4)(x+5)\neq 0$$

$$\therefore x-4\neq 0\quad or\quad x+5\neq 0$$

$$x\neq 4\quad or\quad x\neq -5$$

e)

$$\frac{x-2}{x-1}\div\frac{x-2}{x(x-1)}$$

$$=\frac{x-2}{x-1}\times\frac{x(x-1)}{x-2}$$

$$=x,\quad x\neq 0,1,2$$

Restrictions on original denominator (before it was flipped) also need to be considered. Hence x≠0 is also a restriction

2.3 – Multiplying Rational Expressions Practice Questions

1. Simplify the following expressions and state ***any restrictions***.

a) $\dfrac{3x^3}{2y^2}\times\dfrac{8y^3}{9x}$ b) $\dfrac{-4x}{7y^3}\div\dfrac{-8x^4}{7}$ c) $\dfrac{3}{x-2}\times\dfrac{x-2}{6}$

d) $\dfrac{3(x+2)}{x-1}\div\dfrac{x+2}{x-1}$ e) $\dfrac{6a^3}{a+3}\times\dfrac{5a+15}{8a^3}$ f) $\dfrac{x^2-4}{x+3}\div\dfrac{4x-8}{3x+9}$

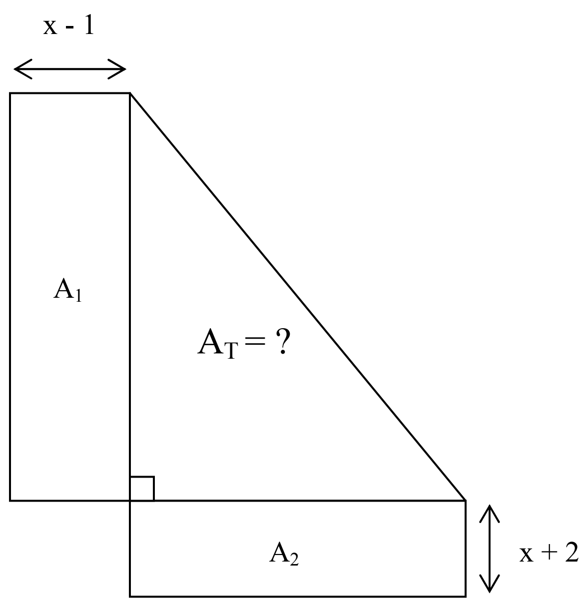
g) $\dfrac{x^2+7x+12}{x^2+4x+4}\times\dfrac{x^2-x-6}{x^2-9}$ h) $\dfrac{2y-3}{3y-1}\times\dfrac{12y^2-19y+5}{4y^2-9}$

i) $\dfrac{x^2+3xy}{x^2-xy-42y^2}\div\dfrac{x^2-9y^2}{x^2-10xy+21y^2}$ j) $\dfrac{x^2-xy-6y^2}{x^2+2xy-8y^2}\div\dfrac{x^2-8xy+15y^2}{x^2-xy-20y^2}$

2. Write a simplified expression for the area of a triangle given the information and diagram below.

$A_1 = 2x^2 + 10x + 12$

$A_2 = 2x^2 - 3x + 1$



Answers 1. a) $\dfrac{4x^2y}{3}, x, y \neq 0$ **b)** $\dfrac{1}{2x^3y^3}, x, y \neq 0$ **c)** $\dfrac{1}{2}, x \neq 2$ **d)** $3, x \neq -2, 1$ **e)** $\dfrac{15}{4}, a \neq -3, 0$ **f)** $\dfrac{3(x+2)}{4}, x \neq 2, -3$

g) $\dfrac{x+4}{x+2}, x \neq -2, -3, +3$ **h)** $\dfrac{4y-5}{2y+3}, y \neq \dfrac{1}{3}, \pm\dfrac{3}{2}$ **i)** $\dfrac{x}{x+6y}, x \neq -6y, \pm 3y, 7y$ **j)** $\dfrac{x+2y}{x-2y}, x \neq -4y, \pm 2y, 3y, 5y$

2. a) $A_t=(x+3)(2x-1)$, x> ½ since measurements need to be positve