

## 2.5 – Rational Functions Introductions

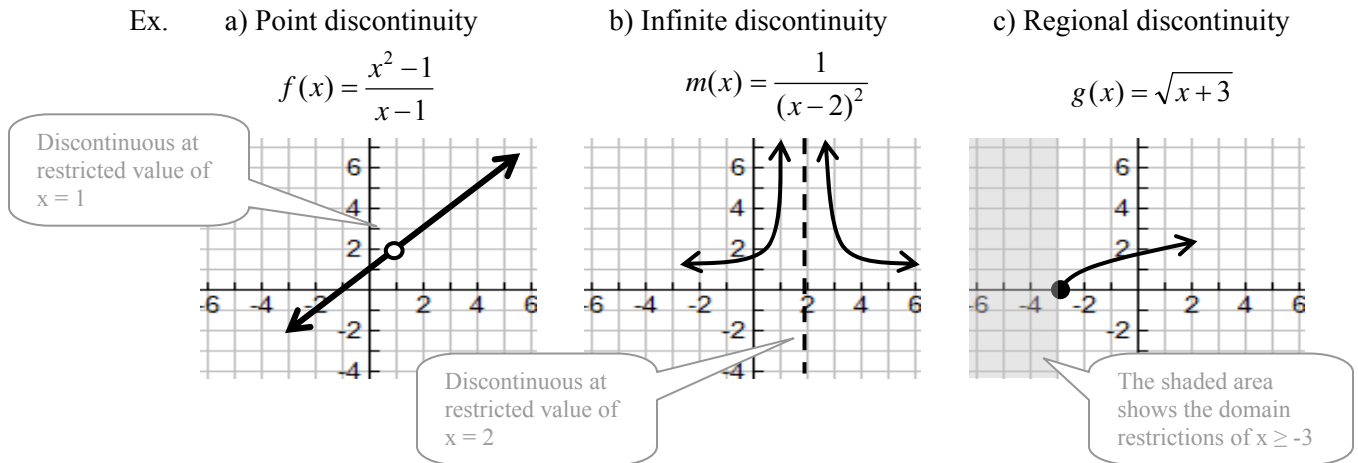
A **rational expression** has the form;  $\frac{a}{b}$ ,  $b \neq 0$

A **rational function** has the form;  $f(x) = \frac{p(x)}{q(x)}$ ,  $q(x) \neq 0$

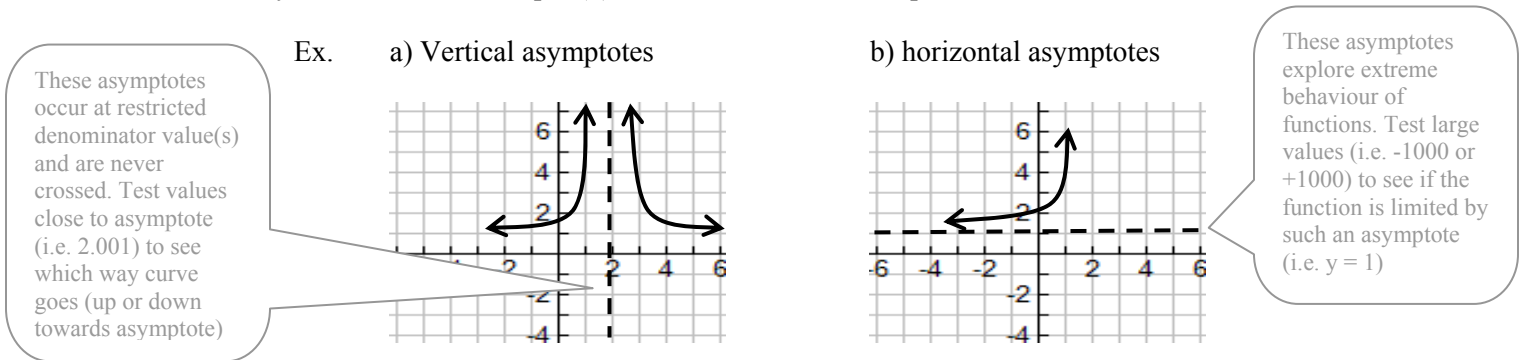
Because the denominator portion of a rational function cannot equal zero, many (but not all) of these functions have domain (x-value) restrictions. These restrictions give rise to points or regions of discontinuity that help define many of the characteristics of these functions.

A continuous function can be completely drawn without lifting your pencil from the page.

A **discontinuous function** has a point or region where the curve breaks (i.e. does not exist).



**Asymptotes** are imaginary lines (shown as dashed lines) that a function might approach but never actually reaches. As in example (b) above these lines can help define a curve.



**Intercepts** (both x and y) also help define function characteristics. Consider  $f(x) = x^2 - 1$

Ex. y-intercept ( $x = 0$ )

$$f(x) = (0)^2 - 1 = 1$$

Substitute  $x = 0$  and solve. As functions only have one y-intercept this is a simple plug and chug operation

x-intercept ( $y = 0$ )

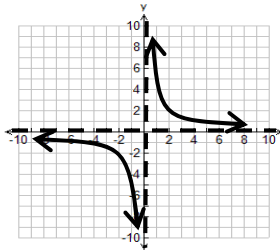
$$0 = x^2 - 1 \\ x = -1 \text{ or } x = 1$$

Solving for zero (x-intercepts) is always more difficult as many algebraic technique(s) like rearranging and factor are often involved.

Although there are several different types of rational functions, we will be studying and transforming two main types. The simplest forms of these two functions are shown below.

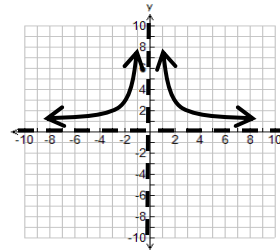
Ex. a) Reciprocal Linear Functions

$$f(x) = \frac{1}{x}$$

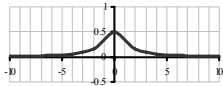


b) Reciprocal Quadratic Functions

$$f(x) = \frac{1}{x^2}$$



Use the graphing calculator to graph the following functions and complete the table to gain insight into some rational functions and their characteristics. You will need to copy the chart into your own notes so that you can fit in adequately sized graphs.

Function	Sketch	Vertical asymptotes (discontinuity)	Horizontal Asymptotes (extreme behaviour)	Intercepts	
				y	x
$f(x) = \frac{1}{x-2}$					
$f(x) = \frac{1}{x} - 2$					
$f(x) = \frac{1}{(x-2)^2}$					
$f(x) = \frac{1}{(x+2)^2}$					
$f(x) = \frac{1}{(x^2-2)}$					
$f(x) = \frac{1}{(x^2+2)}$		n/a	y = 0	1/2	n/a
$f(x) = \frac{x}{x-2}$					
$f(x) = \frac{x^2}{x-2}$					
$f(x) = \frac{x}{(x+2)(x-1)}$					

## 2.5 - Sketching Practice Sheet

