

2.6 – Reciprocal Linear Rational Functions

Basic reciprocal linear rational function has the form; $f(x) = \frac{1}{x}$

Applying transformation we get the general transformations form;

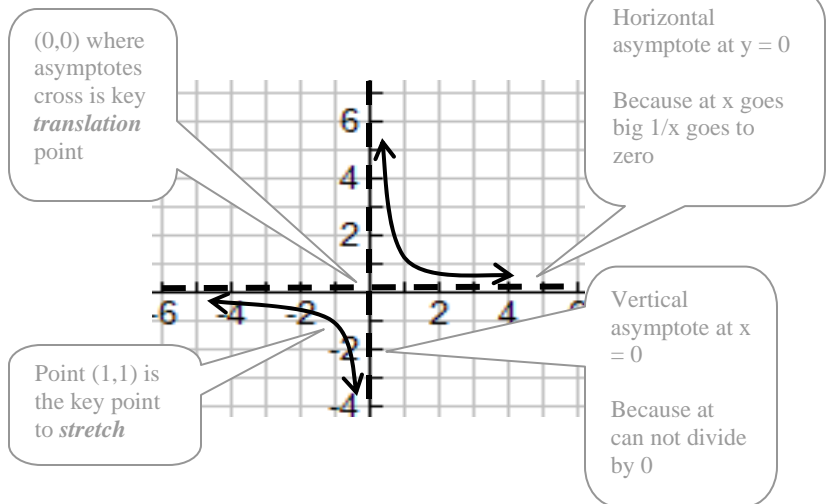
$$f(x) = \frac{a}{k(x-h)} + v$$

where **a** effects vertical stretch and reflection
k effects horizontal stretch and reflection
h effects horizontal translation
v effects vertical translation

To apply transformations effectively we must analyze the base function to note key features like asymptotes, intercepts, vertices (accurate points), and any patterns.

Ex.

$$f(x) = \frac{1}{x}$$



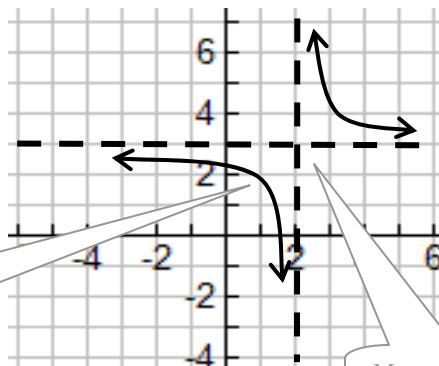
Example 1: Use transformations to sketch the following functions;

a) $f(x) = \frac{-1}{(x-2)} + 3$

| | | | |
|--------------|-------------|------|----|
| Translation: | Horizontal: | +2 | -h |
| | Vertical: | +3 | |
| Stretch: | Horizontal: | n/a | |
| | Vertical: | n/a | |
| Reflection: | Horizontal: | n/a | |
| | Vertical: | flip | |

Because of symmetry the vertical and horizontal flips look the same

1 over and 1 down from asymptote crossing



Move asymptotes together from (0,0)

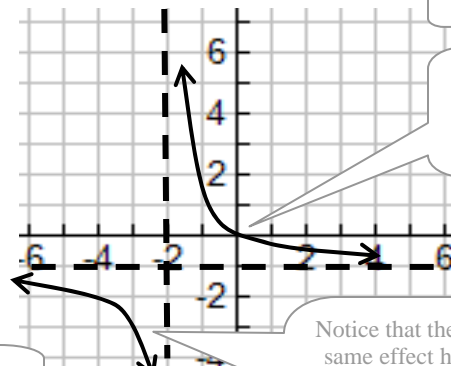
b) $f(x) = \frac{1}{\frac{1}{2}x + 1} - 1$

| | | |
|--------------|-------------|------|
| Translation: | Horizontal: | -2 |
| | Vertical: | -1 |
| Stretch: | Horizontal: | by 2 |
| | Vertical: | n/a |
| Reflection: | Horizontal: | n/a |
| | Vertical: | n/a |

$$f(x) = \frac{1}{\frac{1}{2}(x+2)} - 1$$

Need to re-write to isolate horizontal effect

1/k



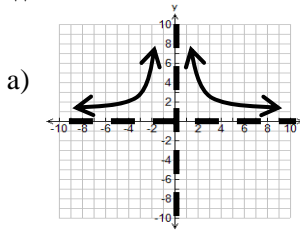
Point now 2 away from asymptote.

Notice that the stretch has same effect horizontally as vertically because

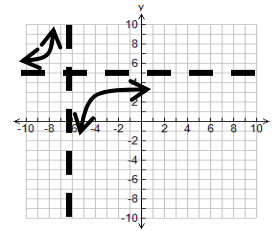
$$\frac{1}{\frac{1}{2}x} = \frac{2}{x}$$

2.6 – Reciprocal Rational Functions Practice Questions

1. Which of the following functions are reciprocal linear rational functions?



b) $f(x) = \frac{1}{2}(x-1) + 3$ c) $f(x) = \frac{1}{x^2 - 1}$



2. Graph the following accurately plotting key points.

a) $f(x) = \frac{1}{x} + 3$

b) $f(x) = \frac{1}{x} + 1$

c) $f(x) = \frac{1}{x-1}$

d) $f(x) = \frac{1}{x+2}$

e) $f(x) = \frac{2}{x}$

f) $f(x) = \frac{1}{2x}$

g) $f(x) = \frac{-1}{x+3} - 2$

h) $f(x) = \frac{1}{-x-3} + 2$

i) $f(x) = \frac{2}{3x}$

j) $f(x) = \frac{1}{2(x+4)} - 1$

k) $f(x) = \frac{1}{2x+4} - 1$

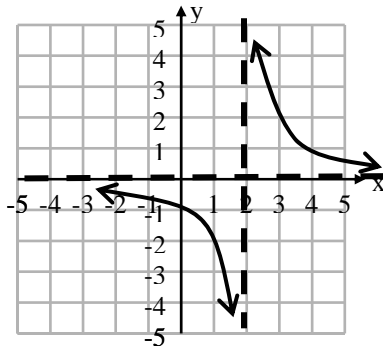
l) $f(x) = \frac{-2}{-3x-6} - 1$

3. Determine the reciprocal linear rational function that best models the following;

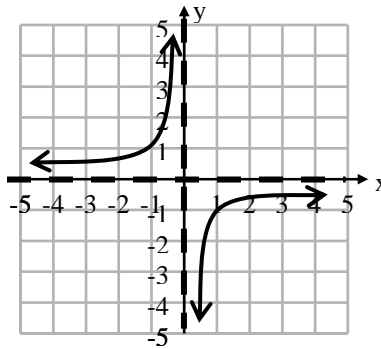
a) horizontal stretch of -2 and vertical transformation of +1

b) $(x, y) \rightarrow (3x - 1, y + 5)$

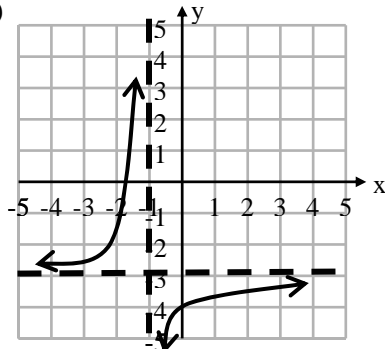
c)



d)



e)



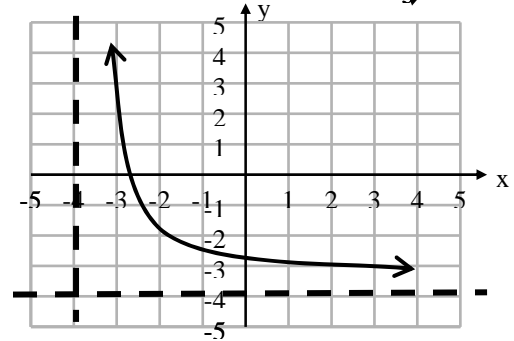
4. Given the linear reciprocal graph below determine;

a) increasing interval(s)

b) decreasing interval(s)

c) slope from i) -3 to -2 ii) -3 to -1
 iii) -3 to 0 iv) -3 to +3

d) slope at i) x = -4 ii) x = -3
 iii) 2 iv) 0 v) +3



Answers 1. d 3. a) $f(x) = \frac{2}{x} + 1$ b) $f(x) = \frac{3}{x+1} + 5$ c) $f(x) = \frac{2}{(x-2)}$ d) $g(x) = -\frac{1}{x}$ e) $y = \frac{-1}{x+1} - 3$ 4. a) none

b) $x < -4$ & $x > -4$ c) i) -5 ii) -2.75 iii) -2 iv) -1 d) i) undefined ii) -10 iii) -1 iv) -1/4 v) -1/10

2.6 - Sketching Practice Sheet

