2.7 – Graphing Rational Functions Using Reciprocal Function Techniques

Going back to the idea that a function performs some sort of operation on a given input, we can think of such operations in isolation.

Ex. A machine's function might be to paint a part red M(red) = ?

A linear function might add 2 f(x) = x + 2

A quadratic's function is to square a number $g(x) = x^2$

The basic functions often get combined. For example you might square and then add. $f(x) = x^2 + 2$

Following this theme, a reciprocal's function is to flip any number or ***any function*** it is given

Ex. $r(x) = \frac{1}{f(x)}$ so given f(x) = 2x -1 then $r(x) = \frac{1}{2x-1}$

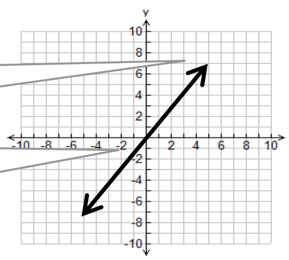
Using this approach we can re-exam what happens to a graph when we take its reciprocal.

See reciprocals properties on following page

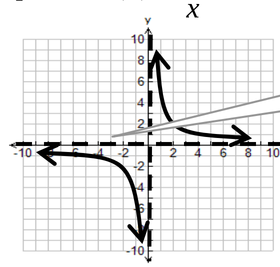
Ex. Given the graph of f(x) = x, then sketch its reciprocal $r(x) = \frac{1}{x}$

Reciprocals of larger numbers means this will get closer to zero

Reciprocals of small numbers means this will go towards infinity



x	y	1/y
-10	-10	-0.1
-2	-2	-0.5
-0.1	-0.1	-10
0	0	n/a
0.1	0.1	10
2	2	0.5
10	10	0.1



Reciprocals of zero is undefined

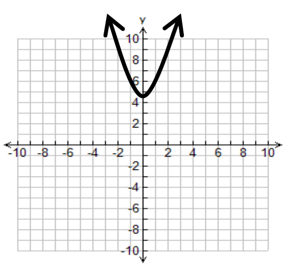
Example 1: Use reciprocal ideas to sketch the following.

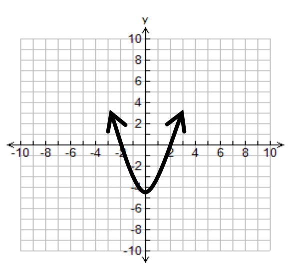
Recognize the reciprocal quadratic form it is written in differs from the one we are comfortable transforming

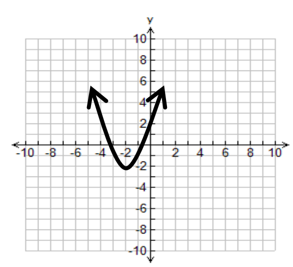
a) $f(x) = \frac{1}{x^2+4}$ b) $g(x) = \frac{1}{x^2-4}$ c) $h(x) = \frac{1}{(x+2)^2-1}$

Sketch the denominator (quadratic translated vertically by +4)

sketch $r(x) = x^2 + 4$ sketch $r(x) = x^2 - 4$ sketch $r(x) = (x+2)^2 - 1$

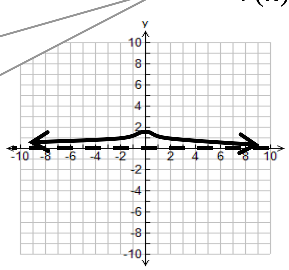


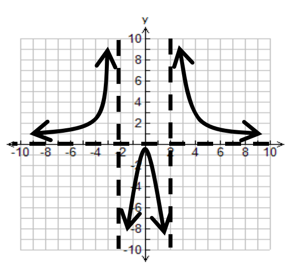


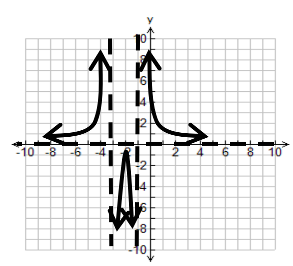


now sketch $f(x) = \frac{1}{r(x)}$

Apply reciprocal properties to r(x). See next page.







2.7 – Graphing Rational Functions Using Reciprocal Techniques Practice Questions

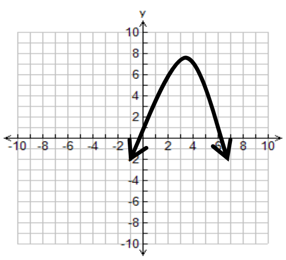
Basic Reciprocal Properties

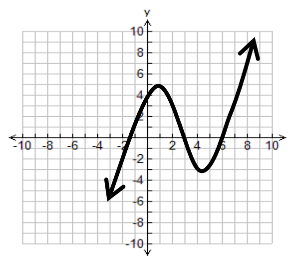
- Reciprocal of 0 is undefined (so vertical asymptote?)
- Reciprocals are same sign as original number $\frac{1}{+ve} = +ve \quad and \quad \frac{1}{-ve} = -ve$
- Reciprocal of 1 is 1
- Reciprocals of very large numbers are close to 0 (so horizontal asymptote)
- Reciprocals of numbers close to 0 are very big (so arrow on end)

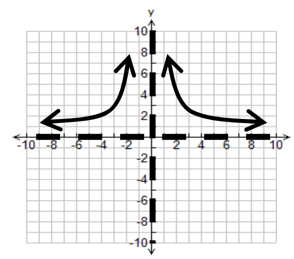
1. Given the function y = f(x), write the equation of the corresponding reciprocal.

a) $y = 3x - 2$ b) $y = 3x^2 + 4x - 2$ c) $y = \sqrt{x-2} + 3$ d) $y = (x+2)^2 - 3$

2. Given the following graph of y=f(x) sketch the reciprocal $r(x) = \frac{1}{f(x)}$

a)


b)


c)


3. Sketch each pair on the same grid.

a) $y = x + 2$ & $y = \frac{1}{x+2}$ b) $y = 2x - 3$ & $y = \frac{1}{2x-3}$ c) $y = 2x^2 + 1$ & $y = \frac{1}{2x^2+1}$

4. Graph each of the following

Which method do you prefer for this question?

a) $y = \frac{1}{3x-2}$ b) $y = \frac{1}{(x-2)^2}$ c) $y = \frac{1}{\frac{1}{2}(x+2)^2}$

There is a difference between e & f. Try to shows this algebraically.

d) $y = \frac{1}{(x-2)^2 + 1}$ e) $y = \frac{1}{-(x-2)^2 + 1}$ f) $y = \frac{-1}{(x-2)^2 + 1}$

g) $m(x) = \frac{1}{(x-2)^2(x+1)}$ h) $y = \frac{1}{(x+2)^2 - 3}$ i) $y = \frac{2}{(x-1)^2 + 3}$

Which method do you prefer for this question?

j) $y = \frac{1}{x^2 - 2} + 1$ k) $y = \frac{1}{x^2 - 6x + 9 + 3} - 2$ l) $g(x) = \frac{1}{(x-2)(x+3)}$

m) $h(x) = \frac{1}{(x-1)^2(x+3)(x-5)}$ n) $f(x) = \frac{1}{\sqrt{x-2} + 3}$ *o) $y = \frac{1}{|x+1| - 3}$

Answers 1. a) $y = \frac{1}{3x-2}$ **b)** $y = \frac{1}{3x^2 + 4x - 2}$ **c)** $y = \frac{1}{\sqrt{x-2} + 3}$ **d)** $y = \frac{1}{(x+2)^2 - 3}$

2.7 - Sketching Practice Sheet