2.8 – Reciprocal Quadratic Rational Functions

Basic reciprocal quadratic rational function has the form; $f(x) = \frac{1}{x^2}$

Unfortunately as this function gets more complicated it becomes more difficult to re-arrange to one form. Hence, we will consider several forms to which we can apply transformations. In another section we will consider sketching by using reciprocal ideas techniques.

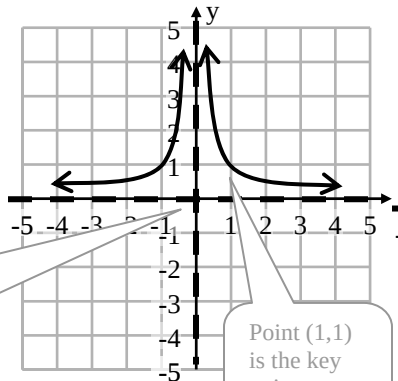
Ex. a) $f(x) = \frac{a}{[k(x-h)]^2} + v$ b) $f(x) = \frac{a}{(x-h)^2 + n} + v$ c) $f(x) = \frac{a}{x^2 - n} + v$

$f(x) = \frac{a}{(x-\sqrt{n})(x+\sqrt{n})} + v$

This is the most straight forward form

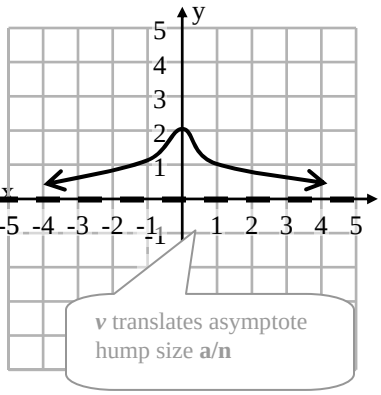
Forms b & c differ based on the sign of ***n***

Difference of squares creates two asymptotes

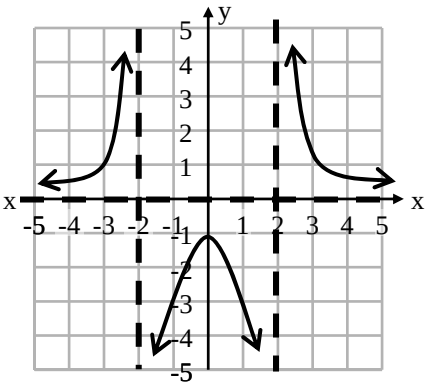


(0,0) where asymptotes cross is key ***translation*** point

Point (1,1) is the key point to ***stretch***



v translates asymptote hump size **a/n**

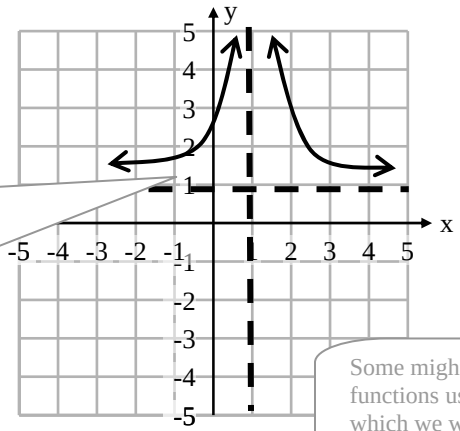


No restrictions (vertical asymptote) as denominator is always a number greater than 0

Example 1: Use transformations to sketch the following functions;

a) $f(x) = \frac{2}{(x-1)^2} + 1$

Translation:	Horizontal:	+1
	Vertical:	+1
Stretch:	Horizontal:	n/a
	Vertical:	by 2
Reflection:	Horizontal:	n/a
	Vertical:	n/a

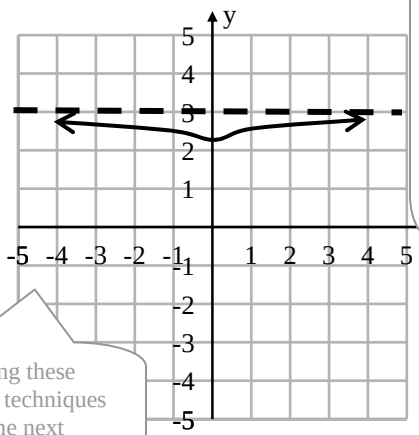


Unlike stretching reciprocal linear functions, the vertical and horizontal stretches must be considered separately with quadratics. Why?

b) $f(x) = \frac{-2}{x^2 + 3} + 3$

As ***h*** gets larger this fraction gets smaller, so smaller hump. This will always occur at x = 0

Translation	H:	no
	V:	+3
Stretch (hump):	H:	-
	V:	-2/3



Don't confuse the +3 with a horizontal translation. The +3 only affects the hump size as a/n

Some might prefer graphing these functions using reciprocal techniques which we will look at in the next section. It depends on how you see it.

2.8 – Reciprocal Quadratic Rational Functions Practice Questions

1. Graph the following accurately plotting key points (use most appropriate technique?)

a) $f(x)=\frac{1}{(x-2)^2}+1$ b) $f(x)=\frac{1}{(x+2)^2}+1$ c) $f(x)=\frac{1}{(x-2)^2}-1$

d) $f(x)=\frac{1}{(x+1)^2}+1$ e) $f(x)=\frac{2}{(x+1)^2}-3$ f) $f(x)=\frac{-2}{(x+1)^2}-3$

g) $f(x)=\frac{1}{(\frac{1}{2}(x+1))^2}+1$ h) $f(x)=\frac{1}{x^2+1}+1$ i) $f(x)=\frac{1}{x^2+1}+2$

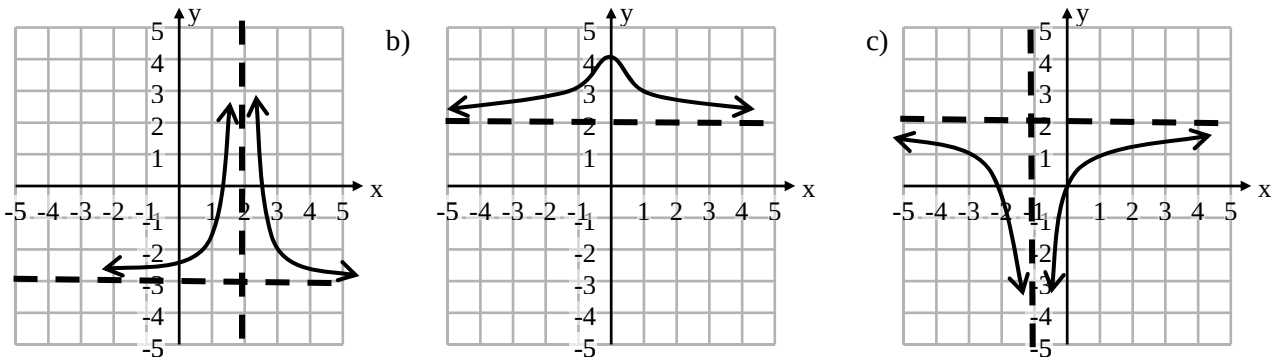
j) $f(x)=\frac{1}{x^2+1}-2$ k) $f(x)=\frac{-1}{x^2+1}+1$ l) $f(x)=\frac{1}{8x^2+1}+1$

m) $f(x)=\frac{1}{(x-3)^2+1}$ n) $f(x)=\frac{1}{x^2+5}$ o) $f(x)=\frac{1}{x^2+\frac{1}{5}}$

p) $f(x)=\frac{5}{x^2+1}$ q) $f(x)=\frac{2}{x^2+\frac{1}{2}}-5$ r) $f(x)=\frac{-3}{x^2+\frac{1}{2}}+2$

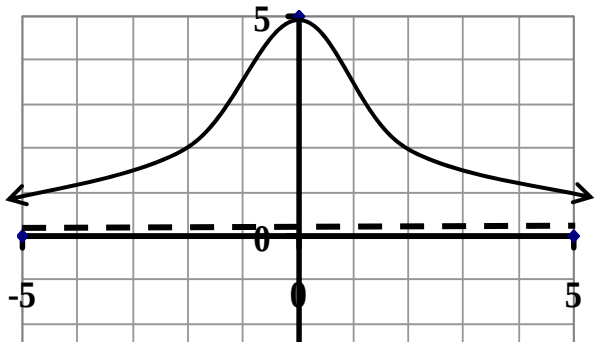
2. Determine the reciprocal quadratic rational function that best models the following;

a) b) c)



3. Given the linear reciprocal graph below determine;

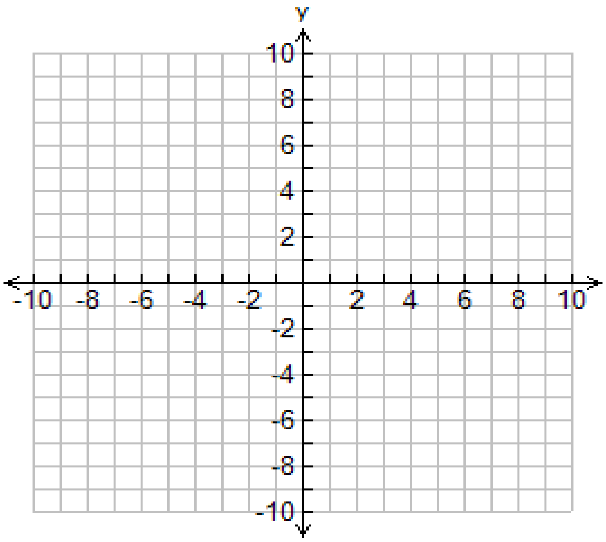
a) increasing interval(s)
b) decreasing interval(s)
c) slope from i) -5 to -4 ii) -5 to 0
iii) -5 to 2 iv) -5 to +5
d) slope at i) x = -5 ii) x = -2
iii) x = 0 iv) x = +2

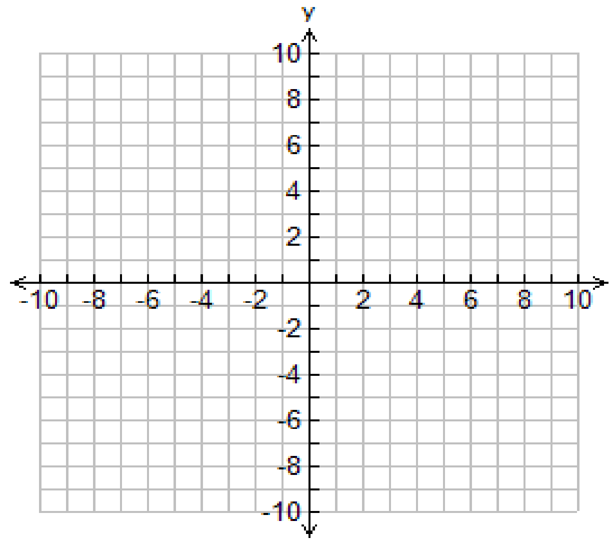


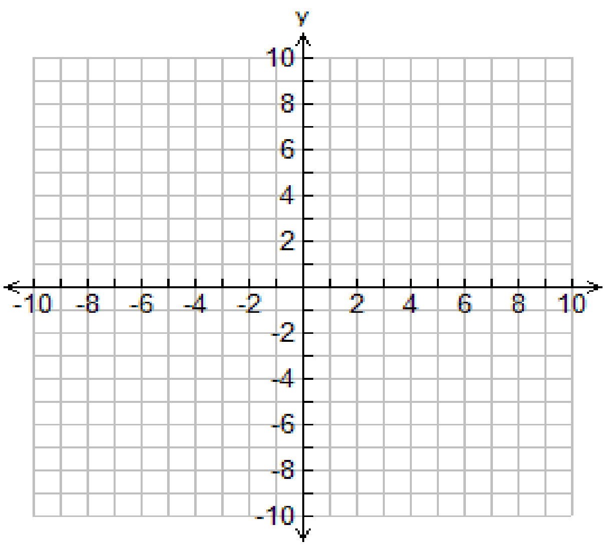
Answers 1. check on calculator **2. a)** $y=\frac{1}{(x-2)^2}-3$ **b)** $f(x)=\frac{2}{x^2+1}+2$ **c)** $g(x)=\frac{-2}{(x+1)^2}+2$

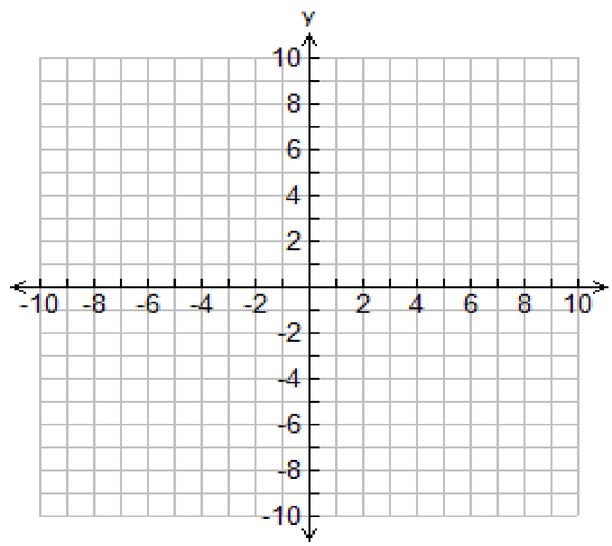
3. a) x<0 **b)** x>0 **c) i)** 0.2 **ii)** 0.8 **iii)** 1/7 **iv)** 0 **d) i)** 0.2 **ii)** 1 **iii)** 0 **iv)** -1

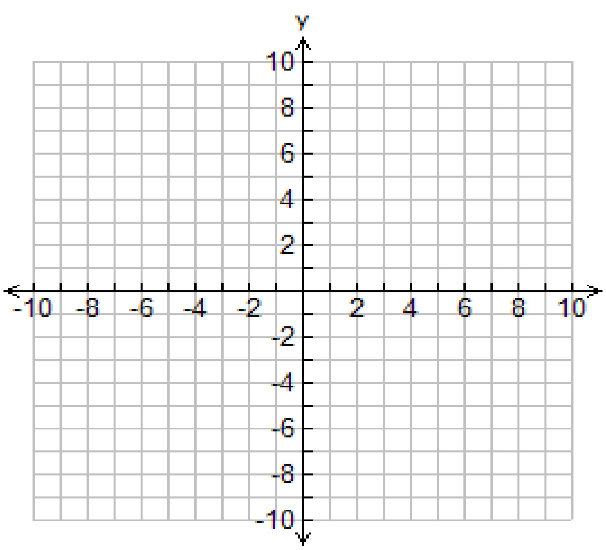
2.8 - Sketching Practice Sheet

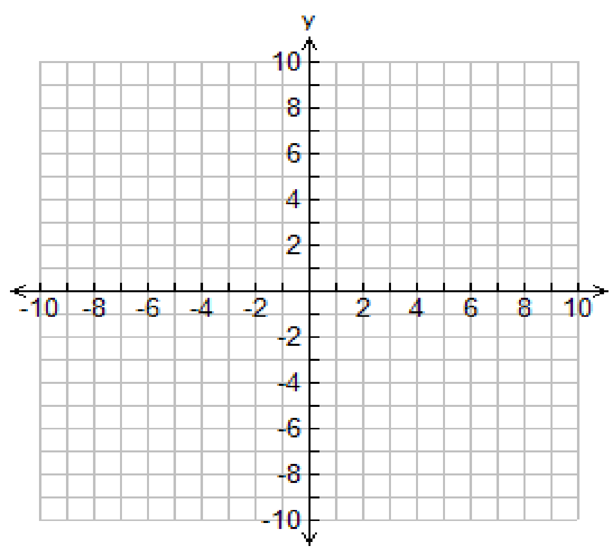












2.7 – reciprocal quadratic function