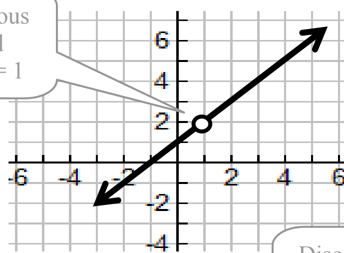
2.9 – Discontinuity

Discontinuity refers to a point or region where a curve breaks (does not exist). This is usually identified as regions where the domain is restricted and occurs in rational functions because the denominator cannot equal zero. We can consider three types of discontinuity.

Ex. a) Point discontinuity

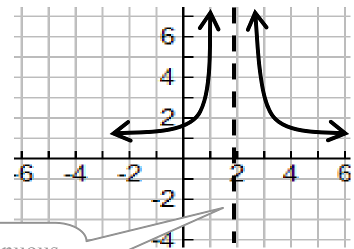
$$f(x)=\frac{x^2-1}{x-1}$$



Discontinuous at restricted value of x = 1

b) Infinite discontinuity

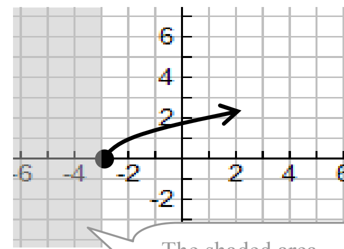
$$m(x)=\frac{1}{(x-2)^2}$$



Discontinuous at restricted value of x = 2

c) Regional discontinuity

$$g(x)=\sqrt{x+3}$$



The shaded area shows the domain restrictions.

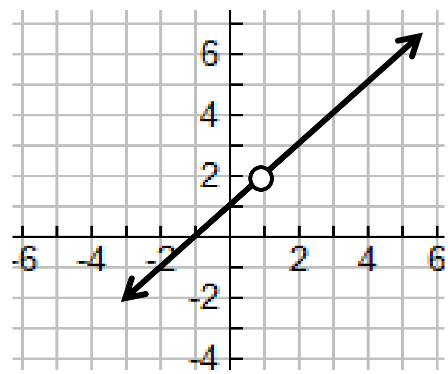
The Hole function is a rational function that can be simplified down to a basic linear function but with a restriction (discontinuity) at one specific point.

Ex.

$$f(x)=\frac{x^2-1}{x-1}$$
$$=\frac{(x-1)(x+1)}{(x-1)}$$
$$=x+1 \quad x\neq 1$$

Function is equivalent to line y = x + 1

Discontinuous at restricted value of x = 1

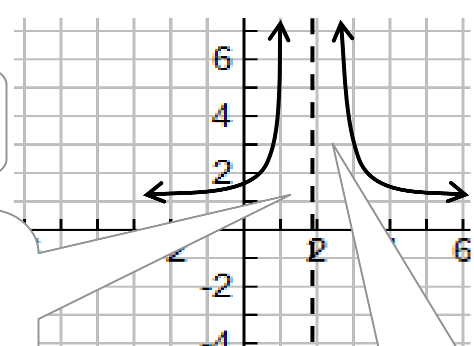


The reciprocal quadratic function is a rational function that can generate a denominator of zero when a value(s) is substituted in. This creates a vertical asymptote in which the value of the function on either side of this discontinuous value approaches infinity.

Ex. $m(x)=\frac{1}{(x-2)^2}, \quad x\neq 2$

The denominator of this function will always be a positive number because it is squared

The function is undefined at x = 2



As we go toward this asymptote at x = 2, the function get very big.

at x = 1.9 m(x) =100
at x = 1.99 m(x) =10000

So curve goes up toward positive infinity.

The same happens on this side of the asymptote.

at x = 2.01 m(x) = 1000

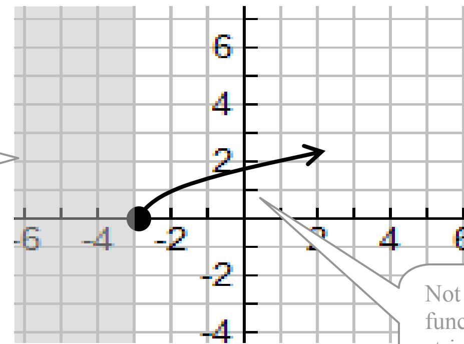
?# ÷ small # = big #
?# ÷ big # = small #

The Radical Function is not really a rational function but as it a discontinous function we will consider it here breifly. Because there are no REAL roots (as opposed to complex roots) of a negative square root any number that generates this case will be restricted and thus create a region of restricted (discontinuous) values.

Ex. $g(x) = \sqrt{x+3}$

$x \geq -3$ otherwise a negative value will be generated under the root sign

The shaded area shows the domain restrictions.



Not a rational function in strictest definition

Example 1: Determine any points of discontinuity.

a) $h(x) = \sqrt{x-1}$

Set what is under root sign to be greater than zero.

$x + 1 \geq 0$

$x \geq -1$

b) $g(x) = \dfrac{2x}{x^2 - 4x - 5}$

Set denominator equal to zero.

$x^2 - 4x - 5 \neq 0$

$(x - 5)(x + 1) \neq 0$

$x \neq 5$ or $x \neq -1$

c) $y = \dfrac{(2x-1)(x+3)}{2x-1}$

simplifies to $y = x + 3$

but still $2x - 1 \neq 0$

$2x \neq 1$

$x \neq ½$

This creates the hole

Example 2: State type of discontinuity for the functions in question #1

a) $h(x) = \sqrt{x-1}$

As restriction describes an interval (i.e. $x \geq -1$)

We know that an entire region is out of bounds

Type of Discontinuity

If using inequalities then regional

If substitution gives $\dfrac{\#}{0}$ then infinite

If substitution gives $\dfrac{0}{0}$ then point

$\therefore$ regional discontinuity

b) $g(x) = \dfrac{2x}{x^2 - 4x - 5}$

Testing either one of the restricted values we see

$$g(5) = \frac{2(5)}{(5)^2 - 4(5) - 5} = \frac{10}{25 - 25} = \frac{10}{0}$$

This suggest a vertical asymptote at x = 5 and x=1

$\therefore$ infinite discontinuity

c) $y = \dfrac{(2x-1)(x+3)}{2x-1}$

Testing the restricted value we see

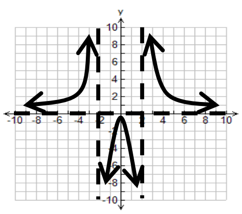
$$y = \frac{\left[2\left(\frac{1}{2}\right) - 1\right]\left[\left(\frac{1}{2}\right) + 3\right]}{2\left(\frac{1}{2}\right) - 1} = \frac{(1-1)(3\tfrac{1}{2})}{1-1} = \frac{0}{0}$$

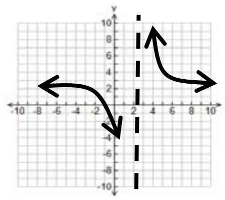
This suggest that this one point (x= ½) can't exist leaving a hole

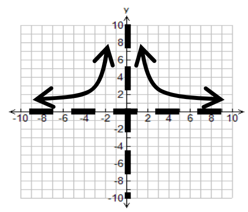
$\therefore$ point discontinuity

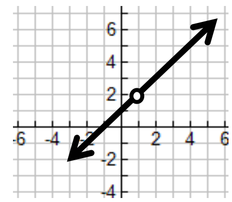
2.9 – Discontinuity Practice Questions

1. Use discontinuity to match the following graphs with the equations below.

a) 

b) 

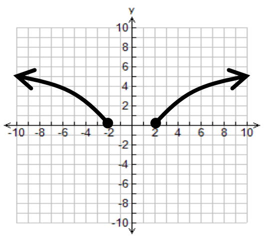
c) 

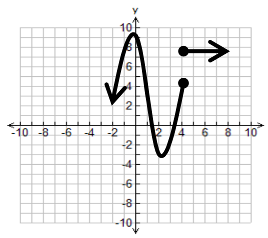
d) 

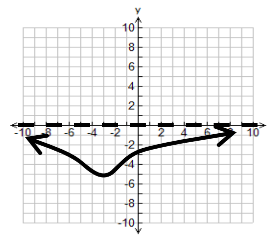
i) $g(x)=\dfrac{2}{(x-1)^2}+1$ ii) $m(x)=\dfrac{1}{x^2-4}$ iii) $f(x)=\dfrac{x^2-1}{x-1}$ iv) $y=\dfrac{x}{x-2}$

2. Identify the type of discontinuity for each graph in question #1.

3. Determine any point(s) of discontinuity for each of the following;

a) 

b) 

c) 

d) $h(x)=\sqrt{x-5}$ e) $f(x)=\dfrac{8x-3}{x}$ f) $h(x)=\dfrac{2x-3}{x^2-9}$

g) $h(x)=\dfrac{2x-3}{x^2-9}+4$ h) $f(x)=\dfrac{x-2}{x^2-4}$ i) $m(x)=\begin{Bmatrix}2x+1, x<0\\ x^2+2, x\geq 0\end{Bmatrix}$

j) $y=\dfrac{(x-3)^2}{x-3}$ k) $m(x)=\dfrac{x^2-3}{x^3}$ l) $h(x)=\sqrt{5-x}$

m) $h(x)=\sqrt{x^2-5}$ n) $y=\dfrac{3x}{x^2-7x+12}$ o) $g(x)=\dfrac{9-x^2}{x-3}$

4. Identify the type of discontinuity for each function in question. #3

Answers 1. a) ii **b)** iv **c)** i **d)** iii **2. a)** infinite **b)** infinite **c)** infinite **d)** point **3. a)** $-2 < x < 2$ **b)** x=4 **c)** continuous
d) x>5 **e)** x=0 **f)** x=-3,3 **g)** x=-3,3 **h)** x=-2,2 **i)** x=0 **j)** x=3 **k)** x=0 **l)** x<5 **m)** $-\sqrt{5}< x<\sqrt{5}$ **n)** x=3,4 **o)** x=3
4.a) regional **b)** point **c)** continuous **d)** regional **e)** infinite **f)** infinite **g)** infinite **h)** point **i)** point
j) point **k)** infinite **l)** regional **m)** regional **n)** infinite **o)** point