

2.11 – Intercepts

Intercepts can occur on both the x-axis and/or y-axis. Consider $f(x) = x^2 - 1$

Ex. y-intercept ($x = 0$)

$$f(x) = (0)^2 - 1 \\ = -1$$

x-intercept ($y = 0$)

$$0 = x^2 - 1 \\ x = -1 \text{ or } x = 1$$

Solving for zero(s) (i.e. x-intercepts) is always more difficult as many algebraic technique(s) like rearranging and factor are often involved.

Substitute $x = 0$ and solve. As functions only have one y-intercept this is a simple plug and chug operation

Example 1: Determine the x and y intercepts of the following, and make rough sketch based on this knowledge.

a) $y = \frac{1}{x+3}$

$$g(x) = \frac{x^2 - 4x - 12}{x - 6}$$

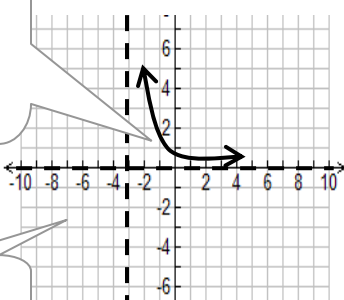
y-intercept ($x = 0$)

$$y = \frac{1}{0+3} \\ y = \frac{1}{3}$$

x-intercept ($y = 0$)

$$0 = \frac{1}{x+3} \\ 0(x+3) = 1 \\ 0 = 1$$

$$\therefore y = 1/3$$



So graph cross y-axis at 1/3

This makes no sense. So graph **does not cross x-axis**. Possible horizontal asymptote?

Graph intercepts then sketch curve through points using asymptotes to help guide

What might happen here?

2.11 – intercepts of rational function

b) $f(x) = \frac{x}{x-2}$

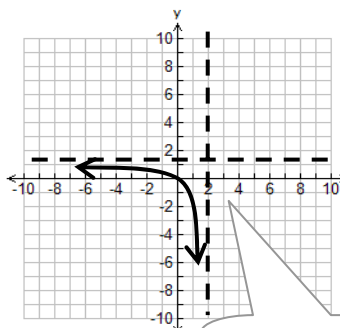
y-intercept ($x = 0$)

$$f(x) = \frac{0}{0-2} \\ = \frac{0}{-2} \\ = 0$$

x-intercept ($y = 0$)

$$0 = \frac{x}{x-2} \\ 0(x-2) = x \\ 0 = x$$

$$\therefore x = 0 \text{ \& } y = 0$$



Try some test values(s) to get shape by y-values on this side of the asymptote

c)

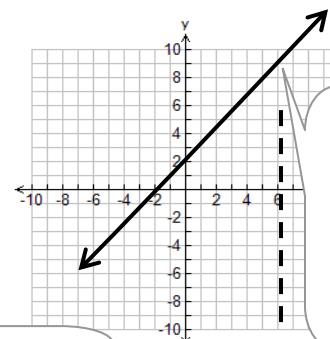
y-intercept ($x = 0$)

$$g(x) = \frac{0^2 - 4(0) - 12}{0 - 6} \\ = \frac{-12}{-6} \\ = 2$$

x-intercept ($y = 0$)

$$0 = \frac{x^2 - 4x - 12}{x - 6} \\ 0 = \frac{(x-6)(x+2)}{(x-6)} \\ 0 = x + 2 \\ -2 = x$$

$$\therefore x = -2 \text{ \& } y = 2$$

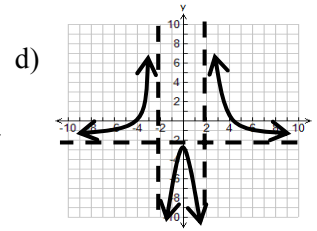
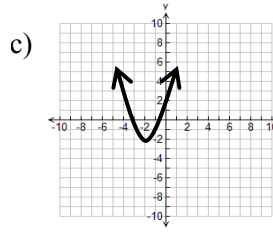
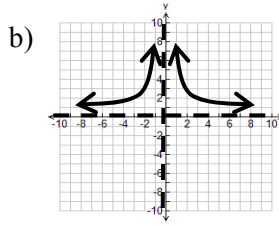
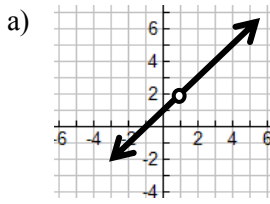


Factor to simplify 1st

Originally might think of vertical asymptote, but later should recognize as hole function

2.11 – Intercepts of Rational Functions Practice Questions

1. State the intercepts of the following graphs.



2. Determine the x and y intercepts for each of the following functions;

a) $f(x) = \frac{1}{x-2}$

b) $g(x) = \frac{x^2 - x}{x-1}$

c) $y = \frac{x}{x-2}$

d) $h(x) = \sqrt{x+9}$

e) $f(x) = \frac{8x-3}{x}$

f) $h(x) = \sqrt{x^2 - 4}$

g) $y = \frac{x^2 - x - 6}{x+2}$

h) $y = \frac{(x-3)^2}{x-3}$

i) $m(x) = \frac{1}{x^2 - 4}$

j) $f(x) = \frac{x-2}{x^2 - 4}$

k) $m(x) = \frac{x^2 - 3}{x^3}$

l) $g(x) = \frac{1-x^2}{x-1}$

m) $y = \frac{3x}{x^2 - 7x + 12}$

n) $h(x) = \frac{x+1}{x-3} - 2$

o) $g(x) = \frac{2}{(x-1)^2} + 1$

3. Use intercepts, asymptotes, your general knowledge about the shape of rational functions, and relevant test values to roughly sketch **b**, **e** & **m** from question #2

Answers 1. a) x=-1 y=1 b) no intercepts c) x=-3, -1 y=2 d) x=-4, 4 y=-3 2. a) y=-1/2 b) x=0 y=0 c) x=0 y=0
d) x=-9 y=+3 e) x=3/8 f) x=-2, 2 g) x=3 y=-3 (hole functions so not vertical at x=-2) h) x=3 y=-3 i) y=-1/4
j) y=1/2 k) x=±√3 y=-3 l) x=-1 y=-1 m) x=0 y=0 n) x=7 y=-7/3 o) x=1±√2 y=3 3. check on your graphing calculator