2.13– Review of Rational Functions

The same operational (+,-,×,÷) rules that apply rational numbers (i.e. fractions) also apply to rational expressions. The only added step is that one must state any restrictions to account for any variables that might be cancelled out during the creating of equivalent expressions.

Ex. Adding or Subtracting
- Find common denominator
- Add/Sub numerator while denominator stays
- Cancel out any common factors to simplify
- State restrictions

Ex. Multiplying or Dividing
- If dividing, flip second term and multiply
- Multiply across
- Cancel out any common factors to simplify
- State restrictions

The rational function has the general form; $f(x)=\dfrac{p(x)}{q(x)},\quad q(x)\neq 0$ Considering the function in the denominator is the result of applying a reciprocal function to some other function, then one might approach rational functions from the point of view of reciprocals.

Ex. Reciprocal Linear Rational Function $r(x)=\dfrac{1}{f(x)}$ where f(x) = 2x + 1 (Linear example)

Reciprocal Quadratic Rational Function $r(x)=\dfrac{1}{f(x)}$ where f(x) = x^2 + 1 (Quadratic example)

This allows one to graph reciprocal functions using two techniques. One involves applying transformations to a known base functions while the other involves using reciprocal ideas on the graph of the function in the denominator.

Ex. $m(x)=\dfrac{1}{(x-2)^2}+3$ is a transformation on $m(x)=\dfrac{a}{(x-h)^2}+v$ (Where a, h & v are know factors)

Ex. $g(x)=\dfrac{1}{x^2-4}$ can be graphed using reciprocal properties on $r(x)=x^2-4$

Basic Reciprocal Properties:
- Reciprocal of 0 is undefined (vertical asymptote?)
- Reciprocal of 1 is 1
- Reciprocals have same sign as original
- Reciprocals of very large numbers are close to 0 (horizontal asymptote)
- Reciprocals of numbers close to 0 are very big (so arrow on end)

In addition to knowing the basic shapes and techniques key features such as intercepts and asymptotes also help to define a curve.

Intercepts describe points where a curve might cross either the x-axis and/or y-axis.

Ex. For y-intercept (x = 0) & x-intercept (y = 0)

Asymptotes are imaginary lines that a function might approach but never actually reaches.

Ex. ***Vertical asymptotes*** occur because of denominator restrictions. They can create a point of discontinuity called infinite discontinuity.

Types of ***discontinuity*** If using inequalities then regional

If substitution gives $\frac{\#}{0}$ then infinite

If substitution gives $\frac{0}{0}$ then point

Recall
?# ÷ small # = big #
?# ÷ big # = small #

So think as follows
?# ÷ small # = ***no asymptote***
?# ÷ big # = ***0***

Horizontal asymptotes occur when a function seems to tend to a certain value at very large positive or negative values. That is as the domain (x-value) approaches infinity what is the function (y-value) equaling. The concept of a ***limit*** can be used to talk about this extreme behaviour.

Given $f(x)=\frac{1}{x}$ explore horizontal asymptotes using form $\lim\limits_{x\to\infty} f(x)$

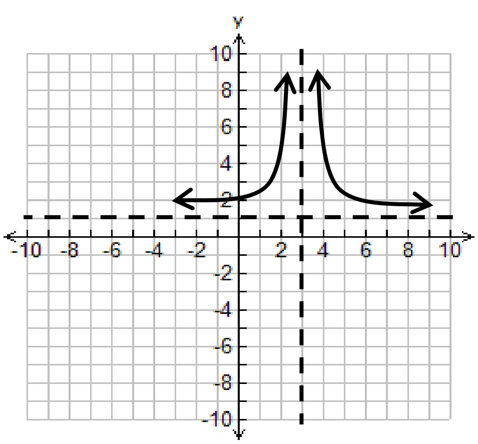
Variable in numerator! Don't recognize type so use intercepts, asymptotes, and test values.

Example 1: Graph the following using the most appropriate technique.

a) $y=\frac{2}{(x-3)^2}+1$

Best done using transformation on the basic quadratic reciprocal.

trans H: +3
V: +1
Stretch H: n/a
V: +2

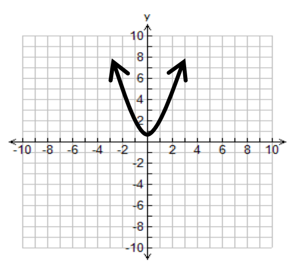


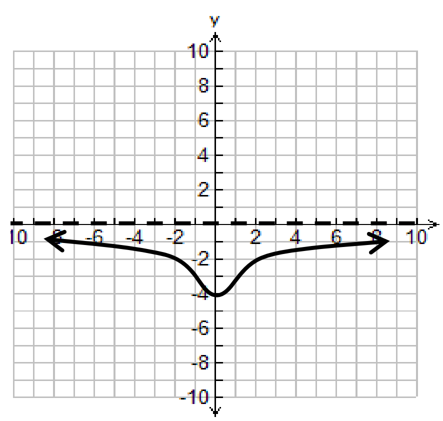
b) $f(x)=\frac{-1}{x^2+\frac{1}{4}}$

Looks like you can use transformations but not quite right form so the best method will be to use reciprocals

graph r(x) = x^2 + ¼

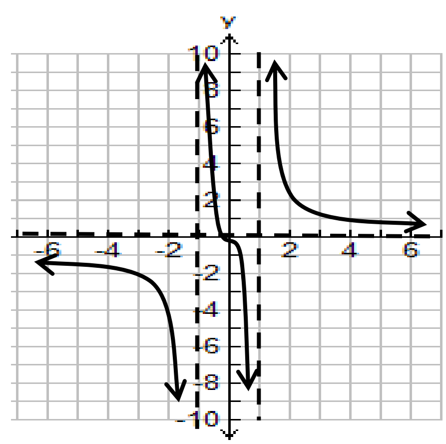
then take reciprocal and flip it





c) $f(x)=\frac{x}{x^2-1}$

Intercepts: x = 0
y = 0
Asymptotes: x = -1, 1
Y = 0



2.13 – Review of Rational Functions Practice Questions

1. Evaluate the following limits;

a) $\lim_{x\to\infty}\frac{9-x^2}{(x-3)^2}$ b) $\lim_{x\to\infty}\frac{5x+3}{x^2-4}+3$ c) $\lim_{x\to\infty}\frac{2x-3}{9-x}+3$

d) $\lim_{x\to\infty}\frac{4x-3x^3}{x^2+2x}$ e) $\lim_{x\to+\infty}\frac{1+5x-3x^2}{3x^2+4x}$ f) $\lim_{x\to-\infty}\frac{5-x}{1+\frac{1}{2}x}+2$

2. Determine any point of discontinuity;

a) $y=\frac{(x-3)^2}{x^2-3}$ b) $m(x)=\frac{x^2+3x}{x}$ c) $h(x)=\frac{x-2}{(x-2)(x+5)}$

d) $h(x)=\sqrt{x^2-4}$ e) $y=\frac{x^2-3x}{x^2-7x+12}$ f) $g(x)=\frac{9-x^2}{(x-3)^2}$

3. Determine intercepts for the following;

a) $y=\frac{x^2+x-6}{x+3}$ b) $y=\frac{(x-2)^2}{x-2}$ c) $m(x)=\frac{1}{x^2-9}$

d) $y=\frac{3x-x^2}{x^2-3x-10}$ e) $h(x)=\frac{3x+1}{x+3}-2$ f) $g(x)=\frac{4}{(x+1)^2}-1$

4. Graph the following using the most appropriate technique (i.e. transformations, reciprocal principles, or intercepts, asymptotes and test values)

a) $f(x)=\frac{-1}{x+2}-3$ b) $f(x)=\frac{1}{\sqrt{x-2}+2}$ c) $f(x)=\frac{2x^2}{x^2-1}$

d) $y=\frac{1}{(x+1)^2-2}$ e) $f(x)=\frac{1}{(x-1)^2}-1$ f) $f(x)=\frac{x^2-4}{x+1}$

g) $f(x)=\frac{1}{-x+3}+2$ h) $f(x)=\frac{2}{(x+1)^2}+3$ i) $g(x)=\frac{x^2+6x+4}{x+3}$

j) $g(x)=\frac{1}{(x-1)(x+4)}$ k) $f(x)=\frac{2}{5x}$ l) $f(x)=\frac{-2}{(x-1)^2}+3$

5. The concentration of a drug in the bloodstream, *x* hours after it is taken orally, is $y=\frac{7x}{x^2+2}$.

a) What is the domain of *y* in this context?
b) What do you know about the graph of *y* just by looking at the equation?
c) Graph the function and describe what happens to concentration over 24 hours.
d) Does the model seem reasonable?

Answers **1. a)** -1 **b)** 3 **c)** 1 **d)** no limit **e)** -1 **f)** 0 **2 . a)** infinite at x=±√3 **b)** point at x=0 **c)** infinite at x=-5,2 **d)** regional -2<x<2 **e)** infinite at x=3,4 **f)** infinite at x=3 **3. a)** x=2(hole), y=-2 **b)** x=2,y=-2 **c)** y= -1/9 **d)** x=0,3,y=0 **e)** x=5,y=1.67 **f)** x=1,-3,y=3 **4. a)** transform linear **b)** reciprocal of radical **c)** intercepts and asymptotes **d)** reciprocal of quadratic **e)** transform quadratic **f)** intercepts and asymptotes **g)** transform linear **h)** transform quadratic **i)** intercepts and asymptotes **j)** reciprocal quadratic **k)** transform linear **l)** transform quadratic **5. a)** concentration as a function of time or C(t) where t>0 **b)** rational, horizontal asymptote y=0, intercepts at origin **c)** function increases to maximum at (1.4,2.5) **d**) seems reasonable

2.13 - Sketching Practice Sheet