3.3 – Quadratic Inequalities

To solve any inequality higher than degree of 1 it is best to interpret and understand them graphically. This is so because one can not use the zero principle to separate the factored form. It no longer holds true for all cases.

> ***Zero Principle*** one of the multipliers must be zero to give zero result

Ex. $(x - 2)(x + 5) = 0$ versus $(x - 2)(x + 5) > 0$

$x - 2 = 0$ or $x + 5 = 0$ $\qquad$ $x - 2 > 0$ or ?

$x = 2$ or $x = -5$

> What will hold true for all values?

Approaching the question ***graphically*** we notice the following;

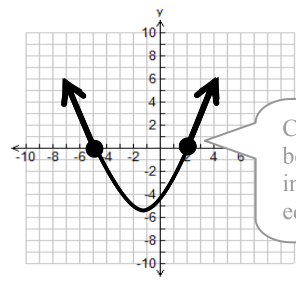
Ex. $(x - 2)(x + 5) \geq 0$ corresponds to the function $f(x) = (x - 2)(x + 5)$ so the question is basically asking when is $f(x) \geq 0$. That is, in what domain interval(s) is the function above the x-axis.

> Say these statements to yourself several times to insure correct sign used

a) Graphing the function $\qquad f(x) = (x - 2)(x + 5)$

b) Highlight intervals

c) Describe interval $\qquad x \leq -5$ or $x \geq 2$



> Closed circle because inequality has equal to sign

Algebraically we can try to solve by considering cases. But this does not always work!

Ex. $(x - 2)(x + 5) \geq 0$ says we need to get the left side to be larger than zero

> So either both brackets are positive or both are negative to give a positive result (i.e. > 0)

Case1: both positive $\qquad$ Case 2: both negative

$(x - 2) \geq 0$ or $(x + 5) \geq 0$ $\qquad$ $(x - 2) \leq 0$ or $(x + 5) \leq 0$

$x \geq 2$ or $x \geq -5$ $\qquad$ $x \leq 2$ or $x \leq -5$

> These statements do not hold true for all value(s) and thus lead to confusion. Stick to **graphing**

Example 1: Solve the following quadratic inequalities.

> Factor to graph zeros

> Rearrange to zero

a) $0 \leq (x + 3)(x - 1)$

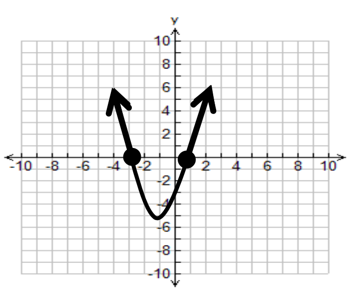
b) $x^2 - x - 6 < 0$

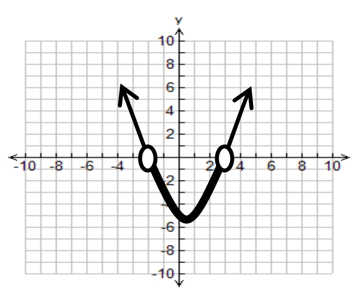
$(x - 3)(x + 2) < 0$

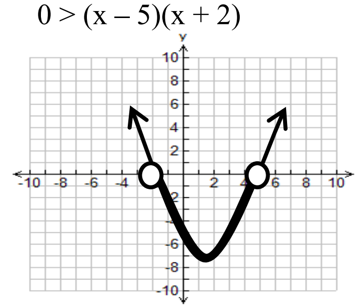
c) $10 > x^2 - 3x$

$0 > x^2 - 3x - 10$

$0 > (x - 5)(x + 2)$







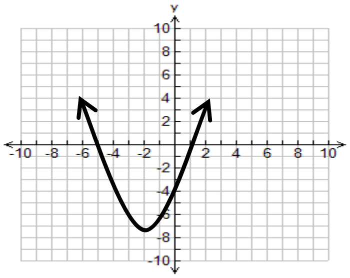
$x \leq -3$ or $x \geq 1$ $\qquad$ $-2 < x < 3$ $\qquad$ $-2 < x < 5$

> **OR** is used as one is describing interval of value(s)

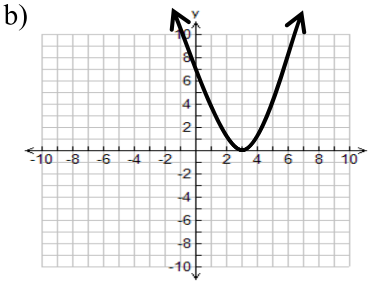
3.3 – Quadratic Inequalities Practice Questions

1. Use graphs below to describe when $f(x) > 0$.

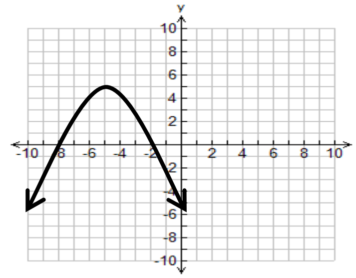
a)



b)



c)



2. Use graphs from questions #1 to describe when $f(x) \leq 0$.

3. Solve the following quadratic inequality by graphing.

a) $(x - 1)(x + 4) < 0$	b) $0 \geq -(x + 3)(x - 1)$	c) $-x^2 - 5x - 6 < 0$
d) $10 < x^2 + 3x$	e) $y^2 > 9$	f) $m^2 - 2m - 24 \leq 0$
g) $10x \geq 25 + x^2$	h) $2x(x - 1) < x^2 + 3$	i) $5x^2 \leq x + 18$
j) $(a - 2)^2 + 3 < 0$	k) $m - 2 \leq -6m^2$	l) $2x^2 > 5x$

4. A ball thrown in the has its height, in meters, described by the function $\boldsymbol{h(t) = -5t^2 + 20t + 2}$, where time is measured in seconds. Determine to the nearest tenth for how long the ball is above 17m high.

Answers 1. a) $x<-5$ or $x>1$ **b)** $x \neq 3$ or $x < 3, 3<x$ **c)** $-8 < x < -2$ **2. a)** $-5 \leq x \leq 1$ **b)** $x=3$ **c)** $x \leq -8$ or $x \geq -2$ **3. a)** $-4<x<1$ **b) x** ≤ -3 or $x \geq 1$ **c)** $x < -3$ or $x > -2$ **d)** $x < -5$ or $x > 2$ **e)** $y < -3$ or $y > 3$ **f)** $-4 \leq m \leq 6$ **g)** $x=5$ **h)** $-1 < x < 3$ **i)** $-1.8 \leq x \leq 2$ **j)** no solution **k)** $-2/3 \leq m \leq 1/2$ **l)** $x<0$ or $x>2.5$ **4.** 2 seconds

3.3 - Sketching Practice Sheets