3.4A – Absolute Value Equations

The ***absolute value function*** performs the operation of converting all numbers to there positive counterpart. So if a number is positive, it leaves it so. If a number is negative it makes it positive.

Ex. absolute value notation: f(x) = |x|

If x = -5 then f(-5) = |-5| = 5 and if x = +5 then f(5) = |5| = 5

So in general we can write: $a(x) = |x| = \begin{cases} -x, \text{ if } < 0 \\ x, \text{ if } \geq 0 \end{cases}$

In physics the difference between vectors and scalars is the associated sign (i.e. direction)

The absolute value function is used when one is interested the ***magnitude of a number*** and not its sign or direction.

Ex. 20 km forward can be represented as +20
20 km in reverse is represented -20

Absolute value of velocity is called speed and is just 20

One is only interested in the speed of the car, direction does not matter.

When solving absolute value equations algebraically one must consider the two cases (positive & negative) that might have originally existed.

Ex. |x – 7| = 4

case 1: original number negative

so -(x – 7) = 4
-x + 7 = 4
3 = x

case 2: original number was positive

so +(x – 7) = 4
x – 7 = 4
x = 11

So entering either 3 or 11 into our original equation will give us a result of 4. Try it!

Example 1: Simplify the following.

a) |-2 – 3| - 6
= |-5| - 6
= 5 – 6
= -1

b) |-3 + 5| - 2
= |+2| - 2
= 2 – 2
= 0

c) 7 - |2 – 5| + 4|- 3|
= 7 - |-3| + 4(3)
= 7 – 3 + 12
= 16

Remember that brackets come in pairs. If you open a bracket the next one must close it

Example 2: Solve the following;

a) |x – 5|=2x – 7

(x – 5) = 2x – 7 or -(x - 5) = 2x – 7
x – 5 = 2x – 7 -x + 5 = 2x – 7
2 = x 12 = 3x
4 = x

This answer is ***inadmissible***. Use substitution or examine the graph to see why?

This is only answer that works.

b) 2x - |x + 3|= 0

2x = |x +3|

2x = (x + 3) or 2x = - (x + 3)
x = 3 2x = -x – 3
x = -1

Isolate and keep a positive sign in front of absolute brackets when splitting for cases

This answer is ***inadmissible***.

Use substitution to check answers

3.4A – Absolute Value Equations Practice Questions

1. Simplify the following.

a) $|-3|$ b) $-|3|$ c) $-|-3|$
d) $|-1-5|-6$ e) $|-3+6|-2$ f) $2-|3-5|+2|-6|$
g) $9-3|2-7|$ h) $|3-6|+|4-1|$ i) $|2-5|-2|-3-1|+|1-6|$

2. Solve the following. Use substitution to check your answers.

a) $|5x| = 10$ b) $|x-5| = 2$ c) $|x| = -7$
d) $|x-4| = 3x-7$ e) $3x-|2x+3| = 0$ f) $|x-5| = 4x+1$

3. Solve the following.

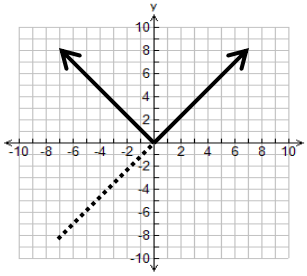
a) $|x^2| = 1$ b) $|x^2-4| = 4$ c) $|x^2-4| = 3$

4. Use a table of values to help graph the following. Check on graphing calculator

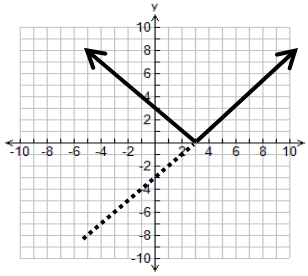
a) $y = |x|$ b) $y = |x-3|$ c) $y = |x|-3$
d) $f(x) = |x^2-4|$ e) $g(x) = |(x+1)(x-2)|$ f) $h(x) = |(x-1)(x+3)(x-5)|$

Answers **1. a)** 3 **b)** -3 **c)** -3 **d)** 0 **e)** 1 **f)** 12 **g)** -6 **h)** 6 **i)** 0 **2. a)** x=-2,+2 **b)** x=3,7 **c)** no solution
d) x=11/4 (x=3/2 is inadmissible) **e)** (x=-3/5 is inadmissible), x=3 **f)** x=4/5 (x= -2 is inadmissible)
3. a) x=-1,+1 **b)** x=0,± √8 **c)** x=±1,±√7

4. a)



b)



c) check others on calculator

On TI-82 use the 2nd ABS button and you will have to add brackets before and after the expression you want to take absolute value of

On TI-83 use MATH, NUM, ABS to apply the function. The calculator starts the brackets and you will have to finish it

3.4A - Sketching Practice Sheets