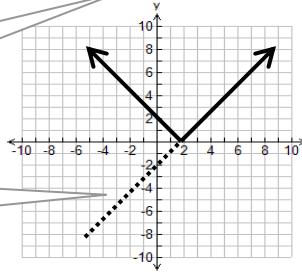


3.4B – Absolute Value Inequalities

Graphically the *absolute value function* reflects any part below the x-axis to the positive side.

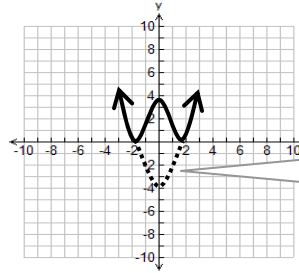
Ex. a) $f(x) = |x - 2|$



Applying absolute value to a **linear** function

Original negative part reflect above x-axis (i.e. positive)

b) $g(x) = |x^2 - 4|$



Applying absolute value to a **quadratic** function

Negative portion is reflected. Dotted not part of graph, just to aid graphing

Before examining inequalities involving absolute value(s) let us re-examine absolute value equations comparing the algebraic and graphically methods.

Ex. $|2x - 1| = 5$

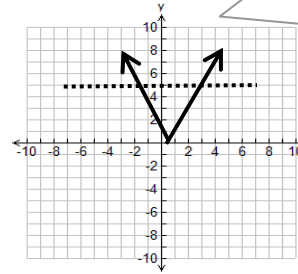
case 1: Negative

$$\begin{aligned} -(2x - 1) &= 5 \\ -2x + 1 &= 5 \\ x &= -2 \end{aligned}$$

case 2: Positive

$$\begin{aligned} +(2x - 1) &= 5 \\ 2x - 1 &= 5 \\ x &= 3 \end{aligned}$$

$|2x - 1| = 5$

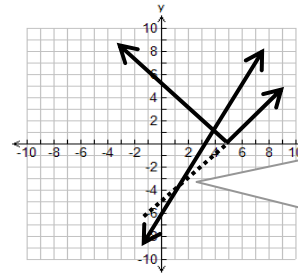


Graphically this asks where does $y = |2x - 1|$ intersect (equal) the $y = 5$ line. It confirms our algebraic solution of an intersection at both $x = -2$, $x = 3$

Ex. $|x - 5| = 2x - 7$

$$\begin{aligned} (x - 5) &= 2x - 7 \\ x - 5 &= 2x - 7 \\ 2 &= x \end{aligned}$$

$$\begin{aligned} \text{or} \quad -(x - 5) &= 2x - 7 \\ -x + 5 &= 2x - 7 \\ 12 &= 3x \\ 4 &= x \end{aligned}$$



The algebra seems to work fine, but one needs to be careful. $x = 2$ is *inadmissible*.

Inadmissible

$\therefore x = 4$ is only solution

Graphically we can graphically see why $x = 2$ is *inadmissible*. It the negative part intersection

With the caveat shown above one should always confirm solutions to absolute value by formally checking using substitution (LS=RS) or by graphing. With absolute value inequalities one will have to graph (sketch) since inequalities are best solved graphically by highlighting the area(s) that match criteria.

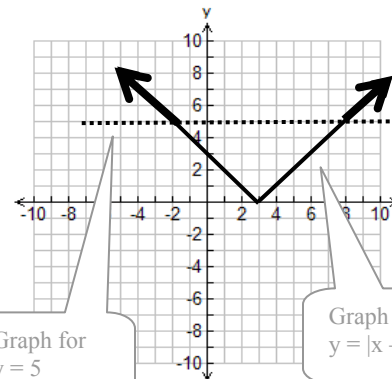
Ex. $|x - 3| > 5$

$$\begin{aligned} -(x - 3) &> 5 & \text{or} & \quad +(x - 3) > 5 \\ -x + 3 &> 5 & & \quad x - 3 > 5 \\ -x &> 2 & & \quad x > 8 \\ x &< -2 & & \end{aligned}$$

Recall sign changes when multiplying by a negative

both solutions confirmed graphically

$\therefore x < -2$ or $x > 8$



Highlight intervals that satisfy conditions

Graph for $y = 5$

Graph for $y = |x - 3|$

3.4B – Absolute Value Inequalities Practice Questions

1. Graph the following.

Transformation is best method for d,e,f. All others should use reflection technique

a) $f(x) = |x + 2|$

d) $h(x) = |x| + 2$

g) $g(x) = |x^2 - 3|$

j) $y = |(x + 3)^2(x - 2)|$

b) $f(x) = |x - 2|$

e) $h(x) = 2|x| - 1$

h) $g(x) = |(x - 3)^2|$

k) $y = |-2x^2 + 4x|$

c) $f(x) = |3x - 1|$

f) $h(x) = \frac{1}{2}|x + 1| - 3$

i) $g(x) = |(x + 2)(x - 1)|$

l) $y = -|(x + 1)^2 - 2| + 3$

2. Solve the following equations algebraically and check graphically

a) $|x + 4| = -2$

c) $3x - |x + 3| = 0$

b) $|x - 4| = 3x - 7$

d) $4x + 1 = |x - 5|$

3. Solve the following inequalities.

a) $|x| < 3$

d) $5 \leq |x - 4|$

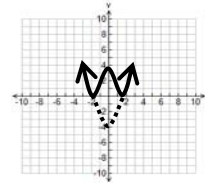
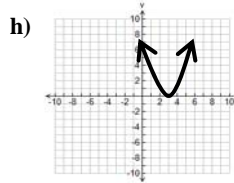
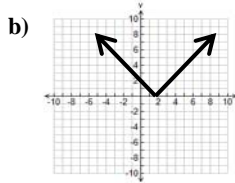
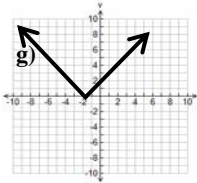
b) $|x - 2| \leq 3$

e) $|x - 4| > 3x - 7$

c) $|x + 3| \geq -1$

f) $2x - |x + 3| < 0$

Answers 1. a&d)



Check other graphs on calculator

2. a) no solution b) $x = 11/4$ c) $x = 1.5$ d) $x = 4/5$ 3. a) $-3 < x < 3$ b) $1 \leq x \leq 5$ c) $x \in \mathbb{R}$ d) $x \leq 1$ or $x \geq 9$ e) $x < 11/4$ f) $x < 3$

3.4 - Sketching Practice Sheets

