3.5A – Dividing Polynomials

Recall long division with whole numbers.

Ex. $579 \div 8$

dividend divisor

$$
\begin{array}{r}
72 \\
8\overline{)579} \\
\underline{56} \\
19 \\
\underline{16} \\
3
\end{array}
$$

dividend

so $579 = 8 \times 72 + 3$

divisor quotient remainder

The same technique can be used with polynomials.

Ex. $(x^2 - 5x - 9) \div (x + 2)$

dividend divisor

Align variables with same degree vertically

$$
\begin{array}{r}
x - 7 \\
x+2\overline{)x^2 - 5x - 9} \\
\underline{x^2 + 2x} \\
-7x - 9 \\
\underline{-7x - 14} \\
+5
\end{array}
$$

dividend

$x^2 - 5x - 9 = (x + 2)(x - 7) + 5$

divisor quotient remainder

Using function notation: $\boxed{\boldsymbol{f(x) = d(x)\,q(x) + r(x)}}$

f(x) = dividend function
d(x) = divisor function
q(x) = quotient function
r(x) = remainder function

Example 1: Perform the following divisions. Express answer in form f(x) = d(x) q(x) + r(x)

a) $(x^3 - x^2 - 4) \div (x - 2)$

Need to put 0x place holder in to keep all terms aligned properly

$$
\begin{array}{r}
x^2 + x + 2 \\
x-2\overline{)x^3 - x^2 + 0x - 4} \\
\underline{x^3 - 2x^2} \\
x^2 + 0x \\
\underline{x^2 - 2x} \\
2x - 4 \\
\underline{2x - 4} \\
0
\end{array}
$$

Since there is no remainder then x-2 must be a factor.

$\therefore\ (x^3 - x^2 - 4) = (x - 2)(x^2 + x + 2)$

b) $(x^4 - 25x^2 + 62x - 36) \div (x^2 + 3x - 18)$

$$
\begin{array}{r}
x^2 - 3x + 2 \\
x^2+3x-18\overline{)x^4 + 0x^3 - 25x^2 + 62x - 36} \\
\underline{x^4 + 3x^3 - 18x^2} \\
-3x^3 - 7x^2 + 62x \\
\underline{-3x^2 - 9x^2 + 54x} \\
2x^2 + 8x - 36 \\
\underline{2x^2 + 6x - 36} \\
+2x
\end{array}
$$

$\therefore (x^4 - 25x^2 + 62x - 36) = (x^2 + 3x - 18)(x^2 - 3x + 2) + 2x$

3.5A – Dividing Polynomials Practice Questions

1. Divide. Express answer in form f(x) = d(x) q(x) + r(x)

   a) $(x^2 - 3x + 5) \div (x - 2)$
   b) $(3x^2 + 2x - 5) \div (x - 2)$
   c) $(3x^2 - 4) \div (x - 4)$
   d) $(m^3 - m^2 + 4m + 15) \div (m^2 + 2m - 3)$
   e) $(x^3 + 3x^2 - 16x + 12) \div (x - 2)$
   f) $(6a^3 + 4a^2 + 9a + 6) \div (3a + 2)$
   g) $(9x^3 - 3x^2 - 4x + 2) \div (x - 2/3)$
   h) $(4x^3 + 32) \div (x + 2)$
   i) $(x^3 - 9x^2 + 26x - 24) \div (x - 2)$
   j) $(-x^3 + 3x + 2) \div (x - 2)$

2. Find the value of ***k*** such that when $2x^3 + 9x^2 + \boldsymbol{k}x - 15$ is divided by x +5 the remainder is zero.

3. When a given polynomial is divided by x – 3, its quotient is $x^2 - 5x - 7$ and its remainder is 5. What is the original dividend polynomial?

4. Find the quotient only; a) $\dfrac{y^3 - 28y - 41}{y + 4}$ b) $\dfrac{4x^3 - 10x^2 + 6x - 15}{2x - 5}$

Answers 1. a) (x-2)(x-1) + 3 **b)** (x-2)(3x+8) + 11 **c)** (x-4)(3x+12) + 44 **d)** $(m^2+2m-3)(m-3) + (13m+6)$
e) $(x-2)(x^2+5x-6)$ or (x–2)(x+6)(x-1) **f)** $(3a+2)(2a^2+3)$ **g)** $(x-2/3)(9x^2+3x-2) + 2/3$
h) $(x+2)(4x^2-8x+16)$ **i)** (x-2)(x-3)(x-4) **j)** (x-2)(-1)(x+1)(x+1) or $-(x-2)(x+1)^2$ **2.** -8 **3.** $x^3-8x^2+8x+26$
4. a) $y^2-4y-12$ **b)** $2x^2+3$