

3.6 – Remainder Theorem

Consider the following comparison when trying to determine the remainder only if dividing $(x^2 - 5x - 9)$ by $(x + 2)$

Method 1: Long Division

$$\begin{array}{r} x-7 \\ x+2 \overline{)x^2-5x-9} \\ \underline{x^2+2x} \\ -7x-9 \\ \underline{-7x-14} \\ +5 \end{array}$$

Because $x + 2 = 0$
We sub in $x = -2$

Remainder
is +5

Method 2: Substituting into function

Given

$$f(x) = x^2 - 5x - 9$$

Sub in $x = -2$

$$\begin{aligned} f(-2) &= (-2)^2 - 5(-2) - 9 \\ &= 4 + 10 - 9 \\ &= +5 \end{aligned}$$

Get same
answer as
remainder

$$\text{Recall } f(x) = d(x)q(x) + r(x)$$

$$\therefore r(x) = +5$$

The remainder theorem highlights this observation by stating that the remainder, when dividing a polynomial by a linear function, can be determined by substituting the solution to the linear term into the polynomial.

If $f(x) \div (x - p)$ gives quotient $q(x)$ and remainder $r(x)$, then $r(x) = f(p)$

Notice
change
in sign

Example 1: Determine remainder only for the following divisions;

a) $(x^3 - 3x^2 + 6x - 5) \div (x - 2)$

if $f(x) = x^3 - 3x^2 + 6x - 5$

then $f(2) = (2)^3 - 3(2)^2 + 6(2) - 5$
 $= 8 - 12 + 12 - 5$
 $= 3$

$$\therefore r(x) = 3$$

b) $(x^3 - 4x^2 + 5x - 1) \div (2x + 3)$

if $g(x) = x^3 - 4x^2 + 5x - 1$

then $g\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right)^3 - 4\left(-\frac{3}{2}\right)^2 + 5\left(-\frac{3}{2}\right) - 1$
 $= \frac{7}{8}$

$$\therefore r(x) = 7/8$$

Example 2: If $(x^3 + 3x^2 + kx + 10) \div (x - 5)$ gives a remainder of 15, find k .

Let $f(x) = x^3 + 3x^2 + kx + 10$ then

$$\begin{aligned} 15 &= f(5) \\ 15 &= (5)^3 + 3(5)^2 + k(5) + 10 \\ 15 &= 125 + 75 + 5k + 10 \\ -39 &= k \end{aligned}$$

$$\therefore k = -39$$

So substituting in
 $x = +5$ gives 15.

The remainder theorem works best for linear divisors and remainder functions that are at most linear terms. For more complex divisions, it is best to go back to the first principles of long division.

3.6 – Remainder Theorem Practice Questions

1. Determine remainder only for each of the following;

a) $(x^2 - 3x + 5) \div (x - 2)$

c) $(3x^2 - 4) \div (x - 4)$

e) $(x^3 + 3x^2 - 16x + 12) \div (x + 1)$

g) $(9x^3 - 3x^2 - 4x + 2) \div (x - 2/3)$

i) $(x^3 - 9x^2 + 26x - 24) \div (x - 5)$

k) $x^3 - x^2 - 4x - 2 \div (x + 2)$

m) $x^3 - 4x^2 + 5x - 1 \div (2x - 3)$

b) $(3x^2 + 2x - 5) \div (x - 2)$

d) $(m^3 - m^2 + 4m + 15) \div (m - 3)$

f) $(6a^3 + 4a^2 + 9a + 6) \div (3a + 2)$

h) $(4x^3 + 32) \div (x + 2)$

j) $(-x^3 + 3x + 2) \div (x - 3)$

l) $x^3 - 4x^2 + 5x - 1 \div (x - 2)$

n) $x^3 + 3x^2 - x - 2 \div (x^2 + 8x + 15)$

2. Find the remainder for; $\frac{3x^5 - 5x^2 + 4x + 1}{(x - 1)(x + 2)}$

3. When $x^4 - kx^3 - 2x^2 + x + 4$ is divided by $x - 3$, the remainder is 16. Find the value of k .

4. When $kx^3 - px^2 - x + 3$ is divided by $x - 1$, the remainder is 4. When the same polynomial is divided by $x - 2$ the remainder is 21. Find the values of k and p

5. For what value(s) of k does the polynomial, $-x^3 + 3x^2 + kx + 1$, have the same remainder when divided by $x + 2$ and $x - 2$?

Answers 1. a) 3 b) 11 c) 44 d) 45 e) 30 f) 0 g) 2/3 h) 0 i) 6 j) -16 k) -6 l) 1 m) 7/8 n) $24x + 73$ 2. $42x - 39$ 3. $k = 2$
4. $k = 3, p = 1$ 5. $k = 4$