

### 3.8 – Graphing Polynomials and the Factor Theorem

Now that we have a way of expressing higher order polynomial functions in factored form we can re-visit graph a polynomial functions in standard form.

Ex. How could one graph

$$f(x) = x^3 + 2x^2 - 5x - 6$$

Using the factor theorem we can re-write as

$$f(x) = (x + 1)(x^2 + x - 6) \\ = (x + 1)(x + 3)(x - 2)$$

In this form we can plot the zeros and use symmetry to sketch the curve.

So far we have been dealing with polynomial functions that have a leading coefficient of 1 in front of their highest degreed term. This simplifies our evaluation process in trying to determine the first factor because we can just substitute in an integer value. When the leading coefficient is an integer other than 1, the possible factors to consider become more complex.

Ex.  $f(x) = 3x^3 + 19x^2 + 27x - 7$

Possible factors are:

$3x - 7$	$3x + 1$	$x - 7$	$x - 1$
$3x + 7$	$3x - 1$	$x + 7$	$x + 1$

Need to test:

$f(7/3)$	$f(1/3)$	$f(7)$	$f(1)$
$F(-7/3)$	$f(-1/3)$	$f(-7)$	$f(-1)$

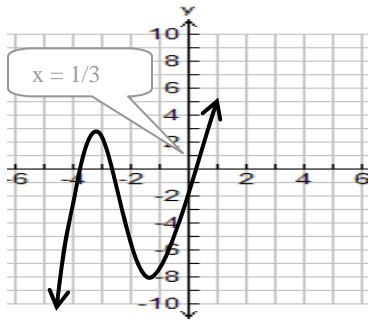
Break each of the end numbers into there factors and then list combinations.

$$3 = (1)(3) \quad -7 = (1)(-7) \\ \text{or } (-1)(-3) \quad \text{or } (-1)(7)$$

So match  $1x$  and  $1 = (x+1)$

Testing fractions will involve more difficult algebra. One can use a graphing calculator to graph the function and then work backwards from the indicated zeros to see which of the above might be a factor.

In this example the graph crosses at  $x = 0.3333$  means  $(3x - 1)$  is a factor



**Example 1:** List possible factors for;

$$g(x) = 5x^3 + 3x^2 - 12x + 4$$

Remember to change the sign

$$5 = (\pm 1)(\pm 5) \\ 4 = (\pm 1)(\pm 4) \\ \text{or } (\pm 2)(\pm 2)$$

$x + 1$	$5x + 4$	$x + 2$	$5x + 2$
$x - 1$	$5x - 4$	$x + 2$	$5x - 2$

$-x + 1$	$-5x + 4$	$-x + 2$	$-5x + 2$
$-x - 1$	$-5x - 4$	$-x + 2$	$-5x - 2$

Use a pattern to account for all possibilities

**Example 2:** Graph the following function.

$$g(x) = 5x^3 + 3x^2 - 12x + 4$$

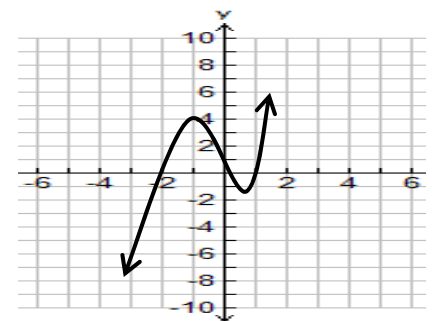
From above we know factors and possible numbers to test. If some easier integral factors (i.e.  $x - 1$ ) present themselves you might take a chance and test these values, otherwise turn to the graphing calculator to help see zero:

Test  $g(1) = 0 \quad \therefore x - 1$  is a factor

Use Synthetic

1	5	3	-12	4
		5	8	-4
	5	8	-4	0

$$5x^3 + 3x^2 - 12x + 4 = (x - 1)(5x^2 + 8x - 4) \\ = (x - 1)(x + 2)(5x - 2)$$



Use **Decomposition** or **Quadratic formula** to continue to factor the quotient

### 3.8 – Graphing Polynomials and the Factor Theorem Practice Questions

1. Sketch the following by hand using the factor theorem to help determine zero(s)/root(s) and check on calculator.

a)  $f(x) = (x - 1)(x + 3)(x - 5)$

b)  $y = x^3 - 7x^2 - 6$

c)  $y = x^3 + 5x^2 + 2x - 8$

d)  $g(x) = x^4 - 2x^3 - 3x^2 + 4x + 4$

2. Using the graphing calculator to help estimate the factors for the following.

a)  $6x^3 + x^2 - 46x + 15$

b)  $6x^3 - 17x^2 + 11x - 2$

c)  $18x^3 - 15x^2 - x + 2$

d)  $4x^4 - 19x^3 + 16x^2 - 19x + 12$

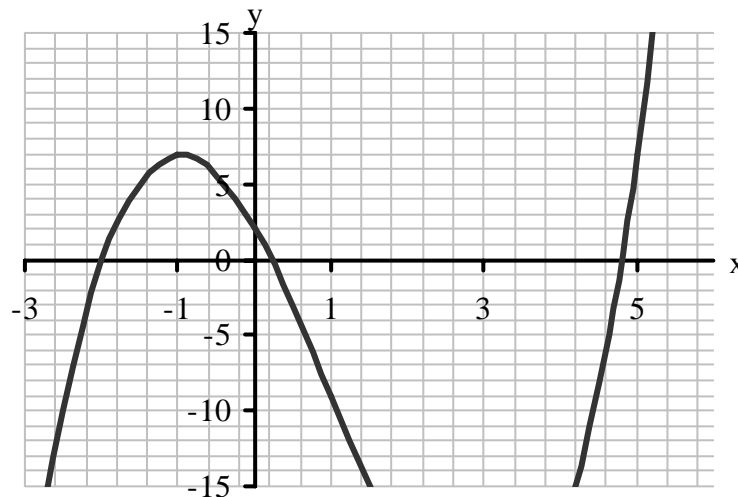
3. Algebraically check/verify using expansion or the factor theorem that your answer to above question work.

4. A third degree polynomial equation has  $x = 2$  as one of its roots. Given  $f(3/2) = 0$ , and  $f(4) = 50$  find the function.

5. Consider the function  $h(x) = x^3 - 3x^2 - 9x + 2$

- Determine the x & y intercepts
- Determine local maxima and minima
- Graph the function
- State increasing intervals
- State decreasing intervals
- State x co-ordinates where function has a slope of zero

**Answers 2. a)**  $(x+3)(3x-1)(2x-5)$  **b)**  $(x-2)(3x-1)(2x-1)$  you will need to zoom in your window to get all the roots otherwise your calculator will only give  $x = \frac{1}{2}$  but you know there must be another to give you the leading co-efficient of 6 **c)**  $(3x+1)(3x-2)(2x-1)$  **d)**  $(x-4)(4x-3)(x^2+1)$  **4.**  $f(x)=(x-2)(2x-3)(x+1)$   
**5. a)**  $x = -2, 0.21, 4.79$  and  $y = 2$  **b)** max is  $y=7$  min is  $y=-25$  **c)** see graph below **d)**  $x < 1$  or  $x > 3$   
**e)**  $-1 < x < 3$  **f)**  $x=3$  &  $x=-1$



### 3.8 - Sketching Practice Sheets

