

3.9 – Oblique Asymptotes

Recall that a rational function may have vertical or horizontal asymptotes that restrict its domain and range and ultimately help to define its shape. Now that we know how to divide polynomials we can examine a third type of asymptote that some rational functions can have.

Oblique asymptotes occur in rational functions when the expression in the numerator is 1 degree higher than the expression in the denominator.

Ex. $f(x) = \frac{p(x)^n}{d(x)^{n-1}}$ $p(x)$ has its degree one higher than $d(x)$

If one actually follows through on the division of $p(x)$ by $d(x)$ a linear quotient will result. This linear quotient represents the line that defines the oblique asymptote. Following from our general division statement one could rewrite a function in such a form to identify the oblique asymptote.

Might think of $\frac{41}{12} = 3 + \frac{5}{12}$

Ex. If $p(x) = d(x)q(x) + r(x)$ then $\frac{p(x)}{d(x)} = \frac{d(x)q(x)}{d(x)} + \frac{r(x)}{d(x)}$ (d(x) will cancel out)

So $f(x) = q(x) + \frac{r(x)}{d(x)}$ where $q(x) = mx + b$ (This will define the oblique asymptote)

Example 1: Determine the oblique asymptote and use this along with other defining characteristics to help sketch the following functions

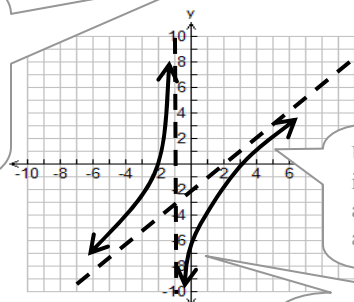
a) $f(x) = \frac{x^2 - x - 6}{x + 1}$

so want to do $(x^2 - x - 6) \div (x + 1)$

use synthetic $-1 \mid \begin{array}{ccc} 1 & -1 & -6 \\ & -1 & 2 \\ \hline 1 & -2 & -4 \end{array}$

so $f(x) = (x - 2) - \frac{4}{x + 1}$

Intercepts: $(0, -6) \text{ \& } (-2, 0), (3, 0)$
 Asymptotes: VA: $x = -1$
 HA: none



These type of functions will never have a horizontal asymptote because numerator is one degree larger

Use test values to see if go towards asymptotes from above or below.

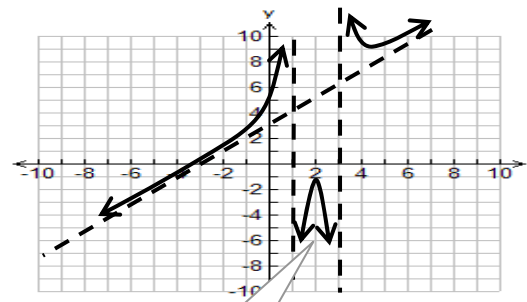
In this example, the intercepts help you figure out which side of the asymptote your line is on.

b) $y = \frac{x^3 - x^2 - 9x + 15}{x^2 - 4x + 3}$

$$\begin{array}{r} x + 3 \\ x^2 - 4x + 3 \overline{) x^3 - x^2 - 9x + 15} \\ \underline{x^2 - 4x^2 + 3x} \\ 3x^2 - 12x + 15 \\ \underline{3x^2 - 12x + 9} \\ 24 \end{array}$$

$y = (x + 3) + \frac{24}{x^2 - 4x + 3}$

Intercepts: $(0, 5) \text{ \& } (-3.1, 0)$
 Asymptotes: VA: $x = 1 \text{ \& } x = 3$
 OB: $y = x + 3$



Try test values At $x = -2, y = -1$

Use long division to avoid mistakes

Need to use graphing calculator to estimate zero

3.9 – Oblique Asymptotes Practice Questions

1. Rewrite each of the following in form $f(x) = q(x) + \frac{r(x)}{d(x)}$ and clearly state oblique asymptote, if it exist.

a) $f(x) = \frac{x^2 - 2}{x}$

c) $g(x) = \frac{x^2 - 6x + 8}{x + 2}$

e) $h(x) = \frac{3x^2 + 2x - 5}{x - 2}$

g) $f(x) = \frac{3x^2 - 2x - 17}{x - 3}$

i) $y = \frac{4x^3 + 32}{x + 2}$

k) $g(x) = \frac{2x^2 + 3x - 1}{x + 1}$

b) $y = \frac{x^2 - 3x + 5}{x - 2}$

d) $m(x) = \frac{x^3 - x^2 + 4x + 15}{x^2 + 2x - 3}$

f) $g(x) = \frac{x^3 + 3x^2 - 16x + 12}{x - 2}$

h) $h(x) = \frac{2x^2 + 9x + 2}{2x + 3}$

j) $y = \frac{x^3 - 1}{x^2 + 2x}$

l) $g(x) = \frac{2x^2 + 3x - 1}{x - 1}$

Compare questions k & l to see differences. Try other examples like this on calculator to see if any pattern?

2. For each question in #1 find any other asymptotes (horizontal or vertical), intercepts, and use test values to complete your analysis of the functions characteristics. Use this information to help sketch the graph.
3. Write the following in rational function format;

a) $g(x) = (x + 3) + \frac{5}{x + 2}$

b) $y = (2x + 1) - \frac{3}{x - 4}$

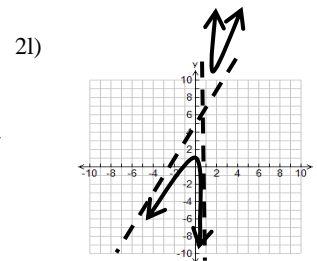
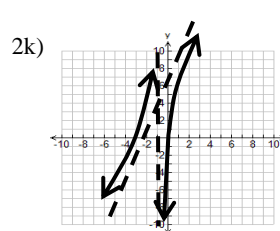
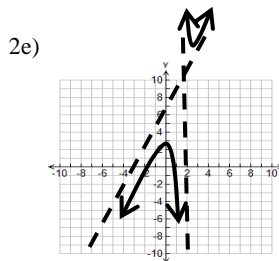
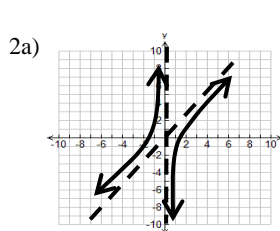
4. Write a rational function in the form $f(x) = \frac{p(x)}{d(x)}$ with oblique asymptote of $y = 2x - 1$

Answers 1. a) $f(x) = x - \frac{2}{x}$ so OA: $y=x$ **b)** $y = (x-1) + \frac{3}{x-2}$ so OA: $y=x-1$ **c)** $y=x-8$ **d)** $y=x-3$ **e)** $y=3x+8$ **f)** none

g) $y=3x+7$ **h)** $y=x+3$ **i)** none **j)** $y=x-2$ **k)** $y=2x+1$ **l)** $y=2x+5$ **2.** some graphs shown below, use calculator to

confirm answers to others **3. a)** $g(x) = \frac{x^2 + 5x + 11}{x + 2}$ **b)** $y = \frac{2x^2 - 7x - 7}{x - 4}$ **4.** answer may vary from this

$f(x) = (2x - 1) + \frac{2}{x + 1}$ becomes $f(x) = \frac{2x^2 + x + 1}{x + 1}$



3.9 - Sketching Practice Sheets

