3.10 – Solving Polynomial Equations

Recall the zero principle when solving quadratic equations.

Ex. $0 = (x + 9)(x - 4)$ So can set $0 = x + 9$ or $0 = x - 4$
And then $-9 = x$ $4 = x$

The same principle can be used to solve higher degree polynomial equations

Ex. $0 = (x - 3)(x + 5)(2x - 3)$ $0 = x - 3$ or $0 = x + 5$ or $0 = 2x - 3$
$3 = x$ $-5 = x$ $3/2 = x$

$\therefore x = -5, 3/2, 3$

Answers usually arranged from low to high

Example 1: Solve the following, where $x \in R$

Real numbers as opposed to complex

a) $0 = (x - 2)(x + 4)(x - 1)$

$0 = x - 2$ or $0 = x + 4$ or $0 = x - 1$
$2 = x$ $-4 = x$ $1 = x$

$\therefore x = -4, 1, 2$

b) $0 = x^3 + 2x^2 - 5x - 6$

Re-write as factors

Test $f(1) = 0$ $\therefore 0 = (x - 1)(x^2 + x - 6)$
$= (x - 1)(x + 3)(x - 2)$

$\therefore x = 1, -3, +2$

Remember to change the sign if you are going to skip zero step

c) $x^3 + 4x = 5$

Re-range first

$x^3 + 4x - 5 = 0$

Factor theorem to find 1st factor

$f(1) = 0$ $\therefore x - 1$ is a factor

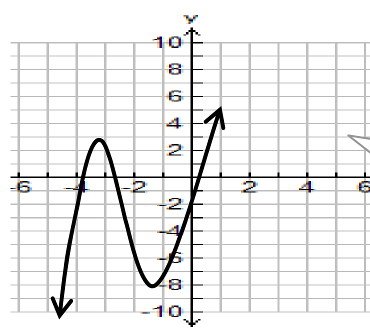
1	1	0	4	-5
		1	1	5
	1	1	5	0

$\therefore (x - 1)(x^2 + x + 5) = 0$

Use formula to see no real roots

$\therefore x = 1$ is only Real solution

d) $f(x) = 3x^3 + 19x^2 + 27x - 7$



Zero's are related to roots of equation. That is where does it cross the x-axis. Hence for complex equations one could graph the corresponding function and estimate the roots.

From graph $\therefore x = -3.7, -2.7$ & $1/3$

e) $4x^4 - 2x^3 = 8x - 16x^2$

$4x^4 - 2x^3 + 16x^2 - 8x = 0$

As with any factoring look to see if there are any common factors you can pull out first.

$2x(2x^3 - x^2 + 8x - 4) = 0$

Sometimes you can recognize special groups. Otherwise use factor theorem technique

$2x[x^2(2x - 1) + 4(2x - 1)] = 0$
$2x(2x - 1)(x^2 + 4) = 0$

$2x = 0$ or $2x - 1 = 0$ or $x^2 + 4 = 0$
$x = 0$ $x = ½$

No real solution
Check $d<0$

3.10 – Solving Polynomial Equations Practice Questions

1. Write a monic polynomial equation whose roots are -1, 2, 5.

Monic indicates that the leading coefficient will be "1". Hence we don't need to consider the various families

2. Write a monic polynomial equation whose roots are -3, 2, ½ and -3/2.

3. Write a polynomial function given x + 1 is a factor and f(-2) = 0, f(3) = 0 and the function goes through the point (2, 24)

4. Solve the following, where $x \in R$

a) $x(x + 2)(x - 5) = 0$
b) $(x - 2)(3x - 4)(x^2 - 1) = 0$
c) $x^2 - 7x + 10 = 0$
d) $x^3 - 4x^2 = 12x$
e) $x^3 - 27 = 0$
f) $x^4 = x^2$
g) $x^3 - 3x^2 - 4x + 12 = 0$
h) $x^3 - 9x^2 = 24 - 26x$
i) $x^3 + 3x^2 = 2$ (use calculator)
j) $6x^4 - x^3 - 56x^2 + 9x + 18 = 0$
k) $5(x + 1)^3 = 5$
l) $x^4 - 7 = 6x^2$

5. Using the graphing calculator to solve the following.

a) $5x^3 - 8x^2 - 27x + 18 = 0$
b) $3x^3 + 19x^2 + 27x = 7$
c) $x^4 + 4 = 0$
d) $2x^3 + 5x^2 + 14x + 6 = 0$
e) $(x - 1)(x + 2) = 4$
f) $x^6 - 3x^3 - 4 = 0$

6. One root of the polynomial equation, $3x^3 - 15x^2 + kx - 4 = 0$, is 2. Determine the value of k and find the other root

Answers **1.** 0=(x+1)(x-2)(x-5) **2.** 0=(x+3)(x-2)(2x-1)(2x+3) **3.** f(x)=-2(x+1)(x+2)(x-3) **4. a)** 0,-2,5 **b)** 2,4/3,1,-1 **c)** -2,-5 **d)** 0,-6,2 **e)** 3 **f)** 0,-1,1 **g)** -2,2,3 **h)** 2,3,4 **i)** -2.7,-1,0.73 **j)** -3,3,-1/2,2/3 **k)** 0 **l)** $\pm\sqrt{7}$ **5. a)** -2,3,3/5 **b)** -3.7, -2.7 & 1/3 **c)** no real solution **d)** -1/2 **e)** -2,3 **f)** 1, $\sqrt[3]{-4}$
6. use f(2)=0 to find k=20. Then synthetic division gives $3x^2 - 9x + 2$