

3.11 – Solving Polynomial Inequalities

Recall that inequalities are best solved graphically. Factoring the function to see the zeros, then highlight and describe areas of the graph that satisfy the inequality.

Ex. $(x - 2)(x + 5) \geq 0$ corresponds to the function $f(x) = (x - 2)(x + 5)$ so the question is basically asking when is $f(x) \geq 0$. That is in what interval is the function above the x-axis.

Say these statements to yourself several times to insure correct sign used

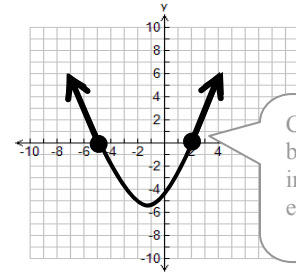
a) Graphing the function

$$f(x) = (x - 2)(x + 5)$$

b) Highlight intervals

c) Describe interval

$$x \leq -5 \text{ or } x \geq 2$$

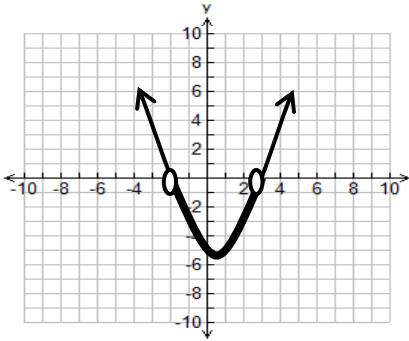


Closed circle because inequality has equal to sign

Example 1: Solve the following inequalities.

a) $x^2 - x + 6 < 0$

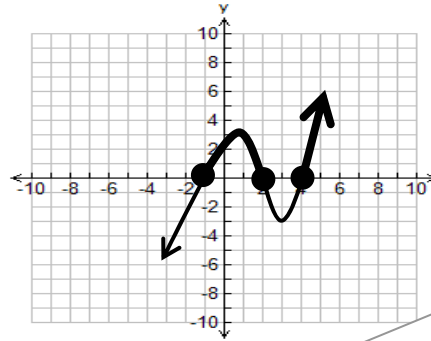
$$(x - 3)(x + 2) < 0$$



$$\therefore -2 < x < 3$$

b) $x^3 - 5x^2 + 2x + 8 \geq 0$

$$(x + 1)(x - 2)(x - 4) \geq 0$$



$$\therefore -1 \leq x \leq 2 \text{ or } x \geq 4$$

By now you know all the factoring steps take place here.

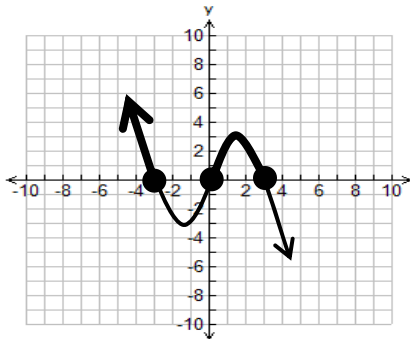
OR is used as one is describing interval of value(s)

c) $9x \geq x^3$

Rearrange to isolate

$$-x^3 + 9x \geq 0$$

$$-x(x - 3)(x + 3) \geq 0$$

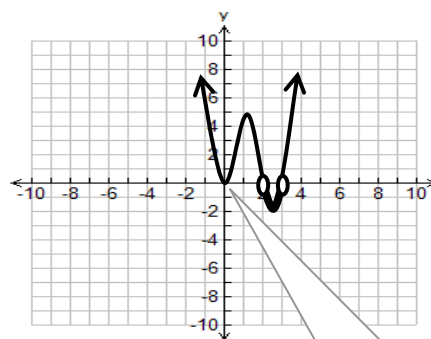


$$\therefore -3 \leq x \text{ or } 0 \leq x \leq 3$$

d) $x^4 < 5x^3 - 6x^2$

$$x^4 - 5x^3 + 6x^2 < 0$$

$$x^2(x - 2)(x - 3) < 0$$

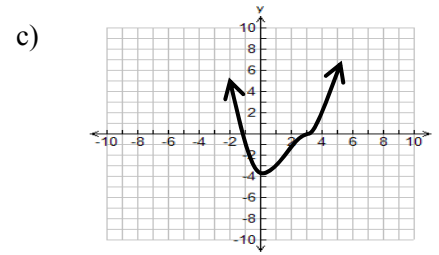
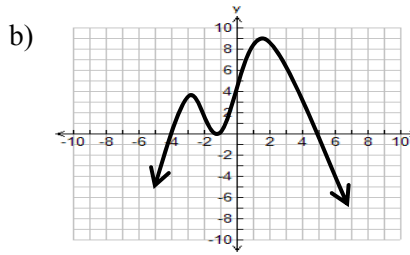
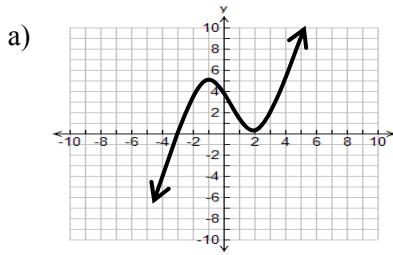


$$\therefore 2 < x < 3$$

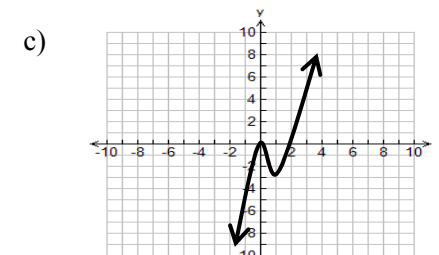
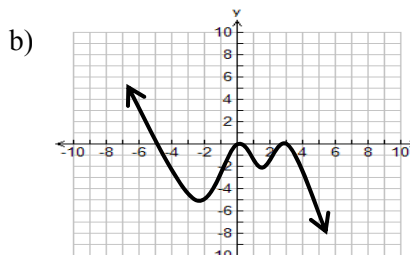
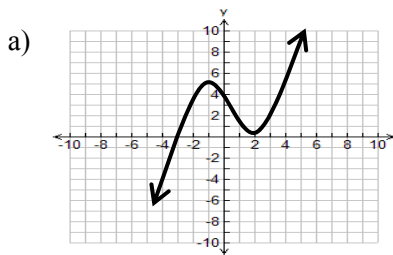
It equals zero at the touch point but is not less than so don't include this point

3.11 – Solving Polynomial Inequalities Practice Questions

1. Describe interval when $f(x) > 0$



2. Describe interval when $f(x) \leq 0$



3. Solve the following inequalities

a) $(x - 2)(x + 1)(x + 2) > 0$

c) $(x - 4)^5(x - 5)^3(x + 3) < 0$

e) $(x - 1)^2(x + 4)^3(x - 2) < 0$

g) $-x^4 - 16 > 0$

i) $x^3 - 5x^2 < x - 5$

k) $-x^4 - 16 < 0$

b) $(x - 3)^2(2x + 1) \leq 0$

d) $(x - 1)(x - 2)(x - 3) \geq 0$

f) $x^4 + 2x^3 + 4x^2 - 7x > 0$

h) $x^3 + 3x^2 < 4$

j) $2x^3 + x^2 - 5x + 2 \leq 0$

4. Write a cubic function $h(x)$, that has a y-intercept of -6 and an x-intercept of -3, given that $h(1) = 0$ and $h(x) < 0$ when $x > -3$

Answers 1. a) $-3 < x < 2$ or $x > 2$ **b)** $-4 < x < -2$ or $-2 < x < 5$ **c)** $x < -1$ or $x > 3$ **2. a)** $x \leq -3$ or $x = 2$ **b)** $x \geq -5$ **c)** $x \leq 2$

3. a) $-2 < x < -1$ or $x > 2$ **b)** $x \leq -\frac{1}{2}$ or $x = 3$ **c)** $x < -3$ or $4 < x < 5$ **d)** $1 \leq x \leq 2$ or $x \geq 3$ **e)** $-4 < x < 1$ or $1 < x < 2$

f) $x < 0$ or $x > 1$ **g)** no real solution **h)** $x < 2$ or $-2 < x < 1$ **i)** $x < -1$ or $1 < x < 5$ **j)** $x \leq -2$ or $0.5 \leq x \leq 1$

k) $x \in \mathbb{R}$ **4.** $h(x) = -2(x+3)(x-1)^2$