

3.12 – Polynomial Equations and Inequalities Problems

Example 1: The dimensions of a rectangular prism are 3 consecutive integers. Given the total volume of the box is 24 cm^3 , find the dimension of the box.

Define your variables so others reading your work can follow it.

Let dimension 1 = x
 Then dimension 2 = $x + 1$
 dimension 3 = $x + 2$

and if Volume = $(d1)(d2)(d3)$

then $24 = x(x+1)(x+2)$
 $24 = x^3 + 3x^2 + 2x$
 $0 = x^3 + 3x^2 + 2x - 24$

If the question was in sentence form then answer in sentence form

Given $f(2) = 0 \therefore x - 2$ is a factor

synthetic $2 \begin{array}{r|rrrr} 1 & 3 & 2 & -24 \\ & & 2 & 10 & 24 \\ \hline & 1 & 5 & 12 & 0 \end{array}$

$\therefore 0 = (x - 2)(x^2 + 5x + 12)$

$\therefore x = 2$ is only solution

No real solution to this part

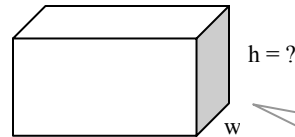
Sub $x = 2$ dimension 2 = $2 + 1 = 3$

Sub in to get other dimensions

Therefore the dimensions of the box are 2cm by 3cm by 4cm.

Example 2: A rectangular solid is to be constructed with a special kind of wire along all of its edges. The length of the base is to be twice the width of the base. The height of the rectangular solid is such that the total amount of wire used for the entire figure is 40cm. Find the range of possible values for the width of the base so that the figure will lie between 2 cm^3 and 4 cm^3 .

Let w = width of base
 l = length of base
 h = height of figure



A diagram can help with variable assignments

We know: $l = 2w$ and if then

$L = 2w$
 Perimeter = $4w + 4l + 4h$
 $40 = 4w + 4(2w) + 4h$
 $40 - 12w = 4h$
 $10 - 3w = h$

So: $h = 10 - 3w$ and if then

Volume = $(w)(l)(h)$
 $= (w)(2w)(10 - 3w)$
 $= 20w^2 - 6w^3$

Apply criteria $2 < V < 4$ so

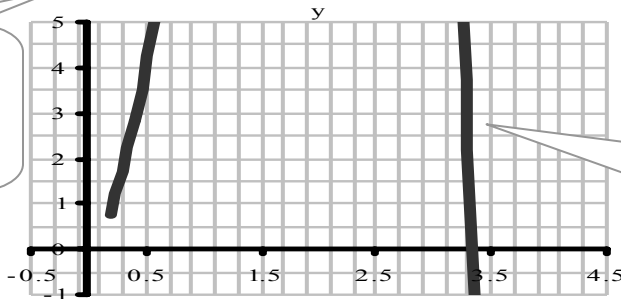
$2 < 20w^2 - 6w^3 < 4$

Graphing the equation and using tracing we find

$0.334 \text{ cm} < w < 0.485 \text{ cm}$

$3.27 \text{ cm} < w < 3.30 \text{ cm}$

Graph corresponding function $V(w) = 20w^2 - 6w^3$
 This gives volume as a function of width



There are two intervals where function is between $2 < v(w) < 4$. Adjust the window so that you can trace and find points accurately.

3.12 – Polynomial Equations and Inequalities Problems Practice Questions

- The height, width and length of a small box are all consecutive integers. If the length and width are increased by 1 and the height is doubled then the volume of the box is increased by 120 m^3 . Find the dimensions of the original box.
- Consider the equation $2x^2 + kx + 9 = 0$. For what value(s) of k is one root double the other?
- The viscosity, v , of oil used in cars is related to its temperature, t , by the formula: $v = -t^3 + 9t^2 - 27t + 21$, where each unit of t is equivalent to 50°C .
 - Graph the function of $v(t) = -t^3 + 9t^2 - 27t + 21$, on your graphing calculator.
 - Determine the value of t for $v > 0$, correct to two decimal places.

- A projectile is shot upwards with an initial velocity of 30 m/s . Its height at time t is given by $h = 30t - 4.9t^2$. During what time interval is the projectile more than 40 m above the ground?
- Solve for x , $x \in \mathbb{C}$

a) $x^4 - 13x^2 + 36 = 0$
 c) $x^6 - 7x^3 = 8$

b) $4x^4 - 2x^3 - 16x^2 = 8x$
 d) $(x^2 - x)^2 - 8(x^2 - x) + 12 = 0$

A tangent line approximates the slope of a curve at the point where it just touches.

- At what point on the graph of $y = x^4 - 20x + 3$ is the tangent to the graph parallel to the line $12x - y - 7 = 0$? Explain.
- We start observing a rocket at time $t = 0$, when it has a velocity of 4 km/s (and its displacement is considered to be zero). Its acceleration is 2 km/s^2 , and this acceleration is increasing at a rate of 0.6 km/s^2 . The displacement of the rocket at t ($t > 0$) is represented by $s(t) = 0.1t^3 + t^2 + 4t$. At what time has the rocket traveled 25 km ?

Answers 1. 3,4,5 2. $k = \pm 9$ 3. a) see graph below b) $t < 59.15^\circ\text{C}$ c) $t > 270.50^\circ\text{C}$ 4. $1.96\text{s} < t < 4.16\text{s}$ 5. a) $x = \pm 2, \pm 3$
 b) $x = 0, \frac{1}{2}, \pm 2$ c) $x = 2, -1, \pm 3i, (1 \pm 2i\sqrt{3})/2$ d) $x = -2, -1, 2, 3$ 6. when $x = 2$ the curve has the same slope of $m = 12$ as the line it is parallel to 7. 3.1s (only intercept from graphing equation $25 = 0.1t^3 + t^2 + 4t$ or $0 = 0.1t^3 + t^2 + 4t - 25$)

