

3.13 – Rational Equations and Inequalities

Rational equations can be solve using the same technique as when solving polynomial equations (i.e. graphically or algebraically) with the added caveat of having to watch for restrictions.

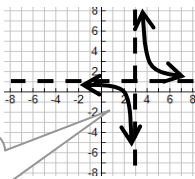
Rational inequalities, like polynominal inequalities, are best solved graphically.

Example 1: Solve the following rational equations

a) $\frac{x-2}{x-3} = 0$

$$x-2=0$$

$$x=2$$



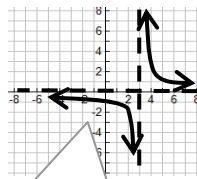
Graphically $x=2$ represents the x-intercept

$1=0$ makes no sense. Graph show that no x-intercept

b) $\frac{1}{x-3} = 0$

$$1 = 0 ?$$

\therefore no solution



c) $1 = \frac{x-2}{x^2+x-6}$

$$x^2+x-6 = x-2$$

$$x^2-4 = 0$$

$$(x-2)(x+2) = 0$$

so $x = 2$ or $x = -2$?

$$\therefore x = -2$$

But if check restrictions would see that $x=2$ is inadmissible

This is only solution

d) $1 = \frac{x-2}{x^2+x-6}$

$$1 = \frac{x-2}{(x-2)(x+3)}$$

$$1 = \frac{1}{x+3}$$

$$x+3 = 1$$

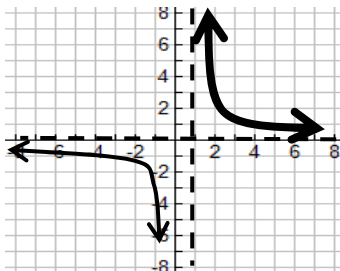
$$x = -2$$

Factoring 1st exposes any restrictions so can see possible inadmissible solutions.

Example 2: Solve the following rational inequalities

a) $\frac{1}{x-1} \geq 0$

graph linear reciprocal and highlight areas ≥ 0

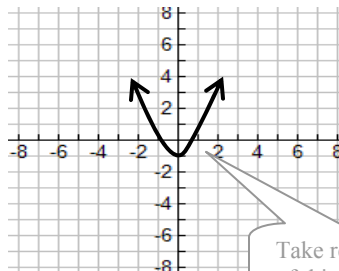


$$\therefore x > 1$$

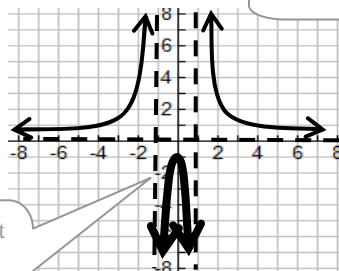
Although the question uses the \geq symbol the value of $x=1$ is restricted so need to use $>$ symbol

b) $\frac{1}{x^2-1} \leq 0$

graph using reciprocal technique on $y = x^2-1$



Take reciprocal of this graph

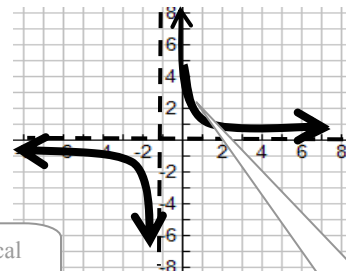


$$\therefore -1 < x < 1$$

Highlight region and describe interval

c) $\frac{1}{x+1} \leq 5$

graph linear reciprocal and highlight areas ≤ 5



$$1 = 5(x+1)$$

$$1 = 5x + 5$$

$$-4 = 5x$$

$$-\frac{4}{5} = x$$

Will need to use algebra (with equal sign) to help get the exact answer for this section

$$\therefore x < -1 \text{ or } x \geq -4/5$$

Need to be careful with signs for these

3.13 – Rational Equations and Inequalities Practice Questions

1. Solve for unknown

a) $\frac{x}{2} + \frac{x}{3} = 5$

b) $-\frac{2}{3}x - 4 = -\frac{4}{5}$

c) $\frac{x+1}{2} = 3$

d) $\frac{2x+1}{3} = -5$

e) $\frac{3}{2} = \frac{7}{x}$

f) $0 = \frac{2}{x-1} - 1$

g) $\frac{x-5}{x+2} = 2$

h) $\frac{x+1}{(x+1)(x-2)} = -1$

i) $\frac{2}{x-1} = \frac{3}{x+2}$

j) $\frac{x-1}{x^2+2x-3} - 2 = 0$

k) $\frac{3}{4x^2} = \frac{5}{3x^3}$

l) $\frac{x}{x^2-6} = 1$

m) $\frac{1}{x-1} - \frac{2}{x+3} = 2$

2. Solve the following inequalities.

a) $\frac{1}{x+2} > 1$

b) $\frac{1}{x^2-4} \leq 0$

c) $\frac{x}{x^2-6} \geq 1$

d) $\frac{2}{x-1} + 3 \geq 3$

e) $\frac{2}{x-1} + 3 < 0$

f) $\frac{1}{(x+2)(x-3)^2} > 0$

g) $\frac{2}{(x-1)^2} + 1 < 0$

h) $\frac{5}{x-2} + \frac{3}{2-x} \geq 1$

i) $\frac{x}{2x-4} - \frac{3}{x-6} + 1 \leq \frac{3}{2}$

Answers 1. a) 6 b) -4.8 c) 5 d) -8 e) 14/3 f) 3 g) -9 h) 1 i) 7 j) -5/2 (x=1 is inadmissible) k) 20/9 (x=0 is inadmissible)
l) x=-2,3 m) x=-3.9, 1.4 (use quadratic formula) 2. a) -2<x<-1 b) -2<x<2 c) -√6 < x ≤ -2 or +√6 < x ≤ 3
d) x>1 e) 1/3 < x < 1 f) -2 < x < 3 or x>3 g) no real solution (graph is above x-axis so is never less than zero)
h) 2 < x ≤ 4 i) 0 ≤ x < 2 or x > 6