3.14 – Review of Equations and Inequalities

A linear relation compares two variables to see how they relate. The degree on both variables can not exceed 1. As such, a linear relation is often called a first degree relation. In most cases this relation will also be a function. A vertical line (i.e. x = ?) is the one exception.

An equation sets a given function equal to some value one is interested in solving for. This is often called, "finding the zeros" because equation is re-arrange to this form which is then where the graph will cross the x-axis (i.e. y = 0)

A quadratic equation has a degree of 2. Three possible solutions exist (no solution, 1 solution or 2 real solutions) which one could solve graphically, by factoring or by using the formula.

Polynomial equations can be solved algebriacally by rearranging to factored form or by using the graphing calculator to estimate zeros.

Polynomial expressions can be divided using ***long division***, where $f(x) = d(x)\,q(x) + r(x)$ with f(x) = dividend, d(x) = divisor, q(x) = quotient, and r(x) = remainder

For linear (i.e. d(x) = mx + b) divisors, ***synthetic division*** provides a quicker method for dividing polynomials

The remainder theorem gives one a quick way to determine the remainder function, r(x), when dividing a polynomial by a linear function.

If f(x) ÷ (x - p) gives quotient q(x) and remainder r(x), then ***r(x) = f(p)***

Notice change in sign

The factor theorem extends the remainder theorem to allow one to determine if a given divisor d(x) will be a factor for the given function f(x).

(x - p) is a factor of f(x) if and only if ***f(p) = 0***

Note sign change

The ***absolute value function*** performs the operation of converting all numbers to their positive counterpart. So if a number is positive, it leaves it so. If a number is negative it makes it positive.

An inequality is interested in determining the domain (as an interval of values) when the function satisfies the given scenario. They are best solved graphically by graphing the corresponding function and then highlighting the areas that satisify the scenario so that you can describe the interval(s)

Example 1: Solve the following inequalities

Highlight regions and describe interval

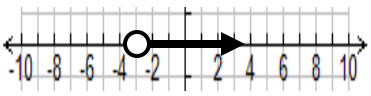
a) $4x < 3 + 5x$

$4x - 5x < 3$

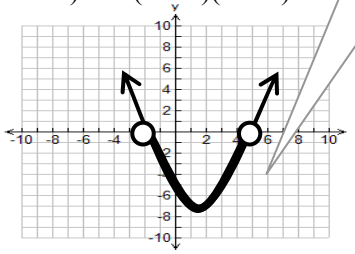
$-x < 3$

$x > -3$

Flip sign when multiply by -1

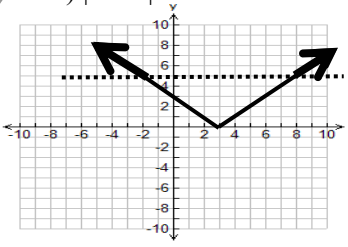


b) $0 < (x - 5)(x + 2)$



$\therefore -2 < x < 5$

c) $|x - 3| > 5$



$\therefore\ x < 2$ or $x > 8$

3.14 – Review of Equations and Inequalities Practice Questions

1. Evaluate; a) $9 - 3|4 - 7|$ b) $|3 - 6| + |4 - 11|$ c) $2 - |3 - 5| + 2|- 6|$
2. Solve for unknown

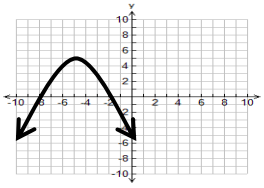
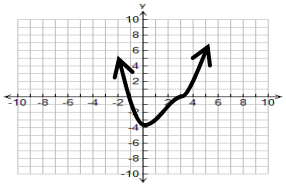
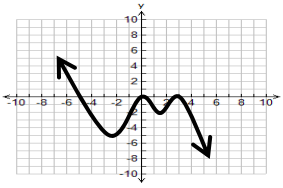
a) $8 + 3x = 4x$ b) $-24 = 8(2y + 5)$ c) $\frac{x+3}{4} = \frac{x+5}{6}$ d) $(2m - 1)(3m + 5) = 0$

e) $24 = x^2 - 2x$ f) $x^3 - 9x^2 = 24 - 26x$ g) $x^4 - 13x^2 + 36 = 0$ h) $(x - 2)(3x - 4)(x^2 - 1) = 0$

i) $x^3 - 27 = 0$ j) $|x - 5| = 2$ k) $|x^2 - 4| = 4$ l) $|x - 4| = 3x - 7$

3. Divide a) $(3x^2+2x-5)\div(x-2)$ b) $(4x^3+32)\div(x+2)$ c) $(x^3+3x^2-16x+12)\div(x+3)$
4. Factor fully; a) $x^3 - 4x + 3$ b) $x^3 + 2x^2 - x - 2$ c) $x^4 - 8x^3 + 3x^2 + 40x - 12$

5. Describe when $f(x) > 0$ and $f(x) < 0$

a)  b)  c) 

6. Graph; a) $f(x) = |3x - 1|$ b) $y = |-2x^2 + 4x|$ c) $g(x) = |(x + 2)(x - 1)|$
7. Solve the following inequalities.

a) $5x + 4x > 18$ b) $2x + 6 \le 6x - 6$ c) $\frac{2+x}{-5} < \frac{2}{3}x - 1$

d) $-x^2 - 5x - 6 < 0$ e) $2x^2 > 5x$ f) $(x - 3)^2(2x + 1) \le 0$

8. Consider the function $h(x) = x^3 - 3x^2 - 9x + 2$
   a) Determine the x & y intercepts b)Determine local maxima and minima
   c) Graph the function d) State increasing intervals
   e) State decreasing intervals f) When does function have a slope of zero

9. Determine oblique asymptotes, if they exist.

a) $y = \frac{x^2 - 3x + 5}{x - 2}$ b) $g(x) = \frac{2x^2 + 3x - 1}{x + 1}$ c) $y = \frac{x^3 - 1}{x^2 + 2x}$

10. Write a rational function in the form $f(x) = \frac{p(x)}{d(x)}$ with oblique asymptote of $y = 2x - 1$
11. Find the value of ***k*** such that when $2x^3 - 3x^2 + kx - 1$ is divided by $x - 1$ the remainder is 2.
12. When a given polynomial is divided by $x - 2$, its quotient is $x^2 - 3x - 7$ and its remainder is -24. What is the original dividend polynomial?
13. When $kx^3 - px^2 - x + 3$ is divided by $x - 1$, the remainder is 4. When the same polynomial is divided by $x - 2$ the remainder is 21. Find the values of ***k*** and ***p***
14. Write a polynomial function through pt(2,24) given $x + 1$ is a factor and $f(-2) = 0$, $f(3) = 0$.
15. A projectile is shot upwards with an initial velocity of 30 m/s. Its height at time t is given by $h = 30t - 4.9t^2$. During what time interval is the projectile more than 40 m above the ground?

Answers 1. a) 0 **b)**10 **c)**12 **2. a)** 8 **b)** -4 **c)** -1 **d)** 1/2, -5/3 **e)** -4,6 **f)** 2,3,4 **g)** -2,2,-3,3 **h)** 2,4/3,1,-1 **i)** 3 **j)** 3,7 **k)** 0,-√8,√8 **l)** 11/4 **3. a)** (x-2)(3x+8)+11 **b)** (x+2)(4x²-8x+16) **c)** (x+3)(x+4)(x-4)+60 **4. a)** (x-1)(x²+x-3) **b)** (x+1)(x+2)(x-1) **c)** (x-3)(x+2)(x²-7x+2) **5. a)** -8<x<-2, x<-8 or x>-2 **b)** x < -1 or x>3, -1<x<3 **c)** x<-5, -5<x<0 or 0<x<3 or x>3 **7. a)** x>2 **b)** x≥3 **c)** x>9/13 **d)** x<-3 or x>2 **e)** x<0 or x>2.5 **f)** x<-1/2 or x=3 **8. a)** x= -2,0.2,4.8 y=2 **b)** max y=7, min y=-25 **d)** x<1 or x>3 **e)** -1<x<3 **f)** x=-1,3 **9.a)** y=x-1 **b)** y=x-8 **c)** y=x-2 **10.** answer may vary **11.** k=4 **12.** x^3-5x^2-x-10 **13.** k=3, p=-1 **14.** g(x)=-2(x+1)(x+2)(x-3) **15.** 1.96<t<4.16

3.14 - Sketching Practice Sheets