4.1 – Review of Exponents and Exponent Laws

Recall the basic definition of exponential form b^x, where base ***b*** is repeatedly multiplied ***x*** times

Ex. $-3^5 = -(3)(3)(3)(3)(3)$ where as $(-3)^5 = (-3)(-3)(-3)(-3)(-3)$

Recall some of the basic exponent conventions are outlined below

Mulitplying exponents with identical bases	$a^n \cdot a^m$	$= a^{n+m}$
Dividing exponents with identical bases	$a^n \div a^m$	$= a^{n-m}$
Power of a power	$(a^n)^m$	$= a^{n \cdot m}$
Power rule	$(ab)^n$	$= a^n \cdot b^n$
Zero exponent	a^0	$= 1$
Negative exponents	a^{-n}	$= \left(\frac{1}{a}\right)^n$ *or* $\frac{1}{a^n}$
Rational exponents	$x^{\frac{a}{b}}$	$= \sqrt[b]{x^a}$ *or* $\left(\sqrt[b]{x}\right)^a$

If you ever forget the time saving rules then just write everything out the long way using the basic definition.

Swing technique?

Memorizing the following chart will facilitate simplification when working with exponents.

base	x^2	x^3	x^4	x^5	x^6
2	$2^2 = 4$	$2^3 = 8$	16	32	64
3	$3^2 = 9$	27	81	243	
4	16	64	256		
5	25	125			
6	36				

Example 1: Simplify the following. Express final answer with positive exponents.

a) $\dfrac{x^3y^2}{x^5y^4}$

$= \dfrac{x^{-2}}{y^2}$

$= \dfrac{1}{x^2y^2}$

Negative becomes positive when switched from above to below or vise versa

b) $\left(5x^2y^{-3}\right)^{-2}$

$= 5^{-2}x^{-4}y^6$

$= \dfrac{y^6}{25x^4}$

c) $\sqrt[5]{32y^{10}}$

$= \sqrt[5]{32} \cdot \sqrt[5]{a^{10}}$

$= 2 \cdot a^{\frac{10}{5}}$

$= 2a^2$

d) $\sqrt{\sqrt{x}}$

$= \left(\sqrt{x}\right)^{\frac{1}{2}}$

$= \left(x^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$= x^{\frac{1}{4}}$

Break it down one step at a time.

Example 2: Evaluate the following as exact answers.

a) $\left(7^3\right)^2 \cdot 7^2 \div 7^5$

$= 7^6 \cdot 7^2 \div 7^5$

$= 7^8 \div 7^5$

$= 7^3$

$= 243$

CAREFUL in that one can only swap if terms are in factored form. Also this only applies to integers or variables. DO NOT confuse with a negative sign in front.

b) $-8^{\frac{-4}{3}}$

$= -\left(8^{\frac{1}{3}}\right)^{-4}$

$= -\left(\sqrt[3]{8}\right)^{-4}$

$= -(2)^{-4}$

$= -\left(\dfrac{1}{2}\right)^4$

$= -\dfrac{1}{16}$

Negative is infront. Leave it there.

Redundant step - which form do you prefer?

c) $\left(\dfrac{25}{4}\right)^{\frac{-3}{2}}$

$= \left(\dfrac{4}{25}\right)^{\frac{3}{2}}$

$= \left(\left(\dfrac{4}{25}\right)^{\frac{1}{2}}\right)^3$

$= \left(\dfrac{2}{5}\right)^3$

$= \dfrac{8}{125}$

Fraction flips Exponent goes positive

d) $\dfrac{2^{-4} + 2^{-6}}{2^{-3}}$

$= \dfrac{\frac{1}{2^4} + \frac{1}{2^6}}{\frac{1}{2^3}}$

$= \left(\dfrac{1}{16} + \dfrac{1}{64}\right) \div \dfrac{1}{8}$

$= \left(\dfrac{5}{64}\right) \times \dfrac{8}{1}$

$= \dfrac{5}{8}$

No rules for add/sub so do long way

4.1 – Exponents Review Practice Questions

1. Express the following as single base with a positive exponent.

a) $4^5 \times 4^{-3}$ b) $3^{-8} \div 3^2$ c) $9^{-7} \div 9^{-6}$ d) $1^{15} \times 1^{-9}$ e) $0.3^{-5} \div 0.3^2$

f) $y^{5\div} y^{-3}$ g) $\frac{2^{-6}}{2^3}$ h) $\frac{12^{-8}}{12^3}$ i) $\left(\frac{2}{3}\right)^4 \div \left(\frac{2}{3}\right)^5$ j) $1^5 \div 1^5$

k) $4^5 \times \frac{4^{-3}}{4^2}$ l) $2^{13} \times 2^{-5} \div 2^6$ m) $3^{-5} \div 3^3 \times 3^{-3}$ n) $7^{15} \div 7^3 \div 7^{-10}$ o) $2^5 \cdot 2^{-3} \cdot 2$

p) $4^5 \times 4^3 \times 4^2$ q) $2^3 \times 2^5 \div 2^6$ r) $x^5 \div x^3 \cdot x^3$ s) $7^{15} \div 7^3 \div 7^{10}$ t) $a^5 \cdot a^3 \cdot a$

u) $\frac{2^3 \cdot 2^4}{2^5}$ v) $4^5 \times \frac{4^3}{4^2}$ w) $\frac{y^5}{y^4} \times y^3$ x) $4^5 \times 4^3$ y) $4^5 \cdot 4^{-3}$

z) $y^3 \div y^{-5}$ aa) $(5x^2)(-2x^7)$ ab) $(x^4)^{-2}$ ac) $(4m^3)^2$ ad) $(x^5)^0$

ae) $(-2x^2)^3$ af) $-(3y^2)^4$ ag) $\sqrt{x\sqrt{x}}$ ah) $x\sqrt{x}$ ai) $\left(\sqrt{4x^2}\right)^3$

2. Evaluate the following as exact answer.

a) $4^3 \times 4^2$ b) $2^{13} \times 2^5 \div 2^6$ c) $3^{-5} \div 3^3 \times 3^3$ d) $7^{15} \div 7^3 \div 7^{10}$ e) $(7^3)^2 \div 7^4$

f) $\frac{2^3 \cdot 2^4}{2^5}$ g) $4^5 \times \frac{4^3}{4^2}$ h) $\frac{5^5}{5^4} \times 5^3$ I) $5^0 \times 5^4$ j) -2^4

k) $\sqrt{-4}$ l) $\sqrt[3]{-8}$ m) 2^{-3} n) $\left(\frac{3}{5}\right)^{-2}$ o) $\left(\frac{4}{9}\right)^0$

p) $2^2 + 2^3$ q) $4^{\frac{1}{2}}$ r) $27^{\frac{1}{3}}$ s) $\sqrt[3]{8}$ t) $8^{\frac{2}{3}}$

u) $\left(\frac{27}{8}\right)^{\frac{2}{3}}$ v) $16^{\frac{3}{4}}$ w) $16^{-\frac{3}{4}}$ x) $(-32)^{\frac{2}{5}}$ y) $-32^{\frac{2}{5}}$

z) $-8^{\frac{-5}{3}}$ aa) $27^{\frac{4}{3}}$ ab) $(2^3)^{-2} \cdot (2^{-2})^{-2}$ ac) $(3^{-1})^3 \times 3^2$ ad) $4^{-2} - 8^{-1}$

ae) $3^2 - 16^{\frac{1}{4}}$ af) $(3^2 + 2^2) \div 8^{\frac{5}{3}}$ ag) $12^0 - 4^{-\frac{1}{2}}$ ah) $\left(4^{-\frac{3}{2}} + 27^{-\frac{2}{3}}\right) \div 16^{-\frac{3}{4}}$

3. Evaluate

a) $3.5^{2.12}$ b) $0.5^{7.2}$ c) $-2.8^{-1.54}$ d) 10.8^{π}

4. Simplify the following.

a) $\left(\frac{x^{-3}}{x^{-1}}\right)^{-2}$ b) $(64x^4)^{\frac{1}{2}}$ c) $\frac{x-9}{x^{\frac{1}{2}}-3}$ d) $\frac{x^{-2}-x^{-3}}{2x}$

Answers 1. a) 4^2 **b)** $1/3^{10}$ **c)** 1/9 **d)** 1^6 **e)** $1/0.3^7$ **f)** y^8 **g)** $1/2^9$ **h)** $1/12^{11}$ **i)** 3/2 **j)** 1 **k)** 1 **l)** 2^2 **m)** $1/3^{11}$ **n)** 7^{22} **o)** 2^3 **p)** 4^{10} **q)** 2^2 **r)** x^5 **s)** 7^2 **t)** a^9 **u)** 2^2 **v)** 4^6 **w)** y^4 **x)** 4^8 **y)** 4^2 **z)** y^8 **aa)** $-10x^9$ **ab)** $1/x^8$ **ac)** $16m^6$ **ad)** 1 **ae)** $-8x^6$ **af)** $-81y^8$ **ag)** $x^{3/4}$ **ah)** $x^{3/2}$ **ai)** $8x^3$ **2. a)** 1024 **b)** 4096 **c)** 243 **d)** 49 **e)** 49 **f)** 4 **g)** 4096 **h)** 625 **i)** 625 **j)** -16 **k)** no solution **l)** -2 **m)** 1/8 **n)** 25/9 **o)** 1 **p)** 12 **q)** 2 **r)** 3 **s)** 2 **t)** 4 **u)** 9/4 **v)** 8 **w)** 1/8 **x)** 4 **y)** -4 **z)** -1/32 **aa)** 81 **ab)** 1/4 **ac)** 1/3 **ad)** -1/16 **ae)** 7 **af)** 17/32 **ag)** ½ **ah)** 17/9 **3. a)** 14.2 **b)** 0.0068 **c)** -0.20 **d)** 1764.4 **4. a)** x^4 **b)** $8x^2$ **c)** $\sqrt{x}+3$ **d)** $(x-1)/2x^4$