

4.2 – Exponential Equations

In an **exponential equation** the unknown is in the exponent part of the power. One can solve such an equation by isolating the power with the unknown exponent on one side of the equation and then trying to re-write the other side of the equation with an identical base. When the bases are similar the exponents can then be set equal

Ex. a) $3^x = 9$
 $3^x = 3^2$
 $3^x = 3^2$
 $\therefore x = 2$

3 to what exponent will give one 9?
 Once bases are identical, set the exponents equal

b) $7^x = 300$
 $7^x = 7^?$
 $7^x = 7^{2.93}$
 $\therefore x = 2.93$

7 to what exponent will give one 300?
 This is more difficult in that no integral exponent works. Various trials are necessary to narrow down to approximate answer.

Although the similar base technique is useful for simple examples and to make a general approximation, example b show the limitation of using the similar base technique. Logarithms provide a way around this. We wil study this in detail later on in the unit.

Ex. $7^x = 300$ taking logarithm of both sides yields $\log 7^x = \log 300$
 Then rearranging one gets $x = \log 300 \div \log 7$

This can solve on a calculator to several places

Example 1: Solve the following.

a) $4^{x+1} = 2^{x-1}$
 Re-write 4 as base 2
 $(2^2)^{x+1} = 2^{x-1}$
 $2^{2(x+1)} = 2^{x-1}$
 Power rule
 $2^{2x+2} = 2^{x-1}$
 $\therefore 2x+2 = x-1$
 $x = -3$

b) $9^{3x+1} = 27^x$
 $(3^2)^{3x+1} = (3^3)^x$
 $3^{2(3x+1)} = 3^{3x}$
 $\therefore 6x+2 = 3x$
 $3x = -2$
 $x = -2/3$

Need to re-write both sides with a common base
 Can distribute as set equations equal

c) $5^{x+1} = 1/125$
 $5^{x+1} = 5^{-3}$
 $\therefore x+1 = -3$
 $x = -4$

Fractions take a bit more thought as negative exponents are required.

d) $3(5^{x+1}) = 15$
 Divide out 3
 $(5^{x+1}) = 15/3$
 $(5^{x+1}) = 5$
 $5^{x+1} = 5^1$
 $\therefore x+1 = 1$
 $x = 0$

0 can be a solution.

e) $3^{x+2} - 3^x = 216$
 $3^x \cdot 3^2 - 3^x = 216$
 $3^x(1 \cdot 3^2 - 1) = 216$
 $3^x(9 - 1) = 216$
 $3^x(8) = 216$
 $3^x = 216/8$
 $3^x = 27$
 $3^x = 3^3$
 $\therefore x = 3$

You can check informally in your head by substituting back into original equation.

Substitute in solution your are checking

formal checking $x = 3$: LS = $3^{3+2} - 3^3$ RS = 216
 $= 3^5 - 3^3$
 $= 243 - 27$
 $= 216$
 $= RS$

Keeps sides separate and show that both work out to the same answer.

A formal check follows specific form to show one that a solution does statidfy the original equation.

$\therefore x = 3$ checks as a solution

Finish with a conclusion statement

4.2 – Exponential Equations Practice Questions

1. Solve the following

a) $2^x = 32$	b) $3^x = 81$	c) $729 = 9^x$	d) $(-3)^x = -27$
e) $(-2)^x = -16$	f) $(-5)^x = 25$	g) $-2^x = -16$	h) $(-1)^x = -1$
i) $(-1)^x = 1$	j) $2^{1-x} = 128$	k) $3^{2x-1} = 1$	l) $4^{x-1} = 4$
m) $6^{x+3} = 6^{2x}$	n) $2^{x+1} = 2^{2x+1}$	o) $8^{3x} = 4^{x+1}$	p) $3^x = 9^{2x}$
q) $(-8)^{1-2x} = (-32)^{1-x}$	r) $4^x = 8^{x+1}$	s) $5^{4-x} = 1/5$	t) $3^{3x-1} = 1/81$
u) $25^{1-3x} = 125^{-x}$	v) $2(3^{x-2}) = 18$	w) $2 = 6(3^{3x-2})$	x) $3 = 27(3^{3x+1})$

2. Solve and check

a) $2^{x+5} + 2^x = 1056$	b) $3^{x+3} - 3^{x+1} = 648$	c) $10^{x+4} = 11 - 10^{x+3}$	d) $5^{x+2} - 5^{x+3} + 2500 = 0$
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3. Solve

a) $\frac{27^x}{9^{2x-1}} = 3^{x+4}$	b) $27^{x+1} = \left(\frac{1}{9}\right)^{2x-5}$	c) $\frac{2^{2x+1}}{2^{x-3}} = 4$	d) $\frac{9^{x+4}}{27^{x-1}} = 81$
e) $\frac{8^{x+2}}{4^{x+3}} = 16^{x-3}$	f) $2^{x^2+2x} = 2^{x+6}$	g) $3^{x^2-2x} = 3^{x-2}$	h) $2^{2x^2-3x} = 2^{x^2-2x+12}$

4. Find x and y if; $\frac{16^{x+2y}}{8^{x-y}} = 32$ and $\frac{32^{x+3y}}{16^{x+2y}} = \frac{1}{8}$

5. A bacteria population doubles every 3 hours. Determine how much time has elapsed if an initial 1200 bacteria population grows to the following.

a) 2400	b) 9600	c) 76800	d) 6400
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6. Strontium-90 has a half life of 28 years. This means that after 28 years only half the amount of strontium will remain. Use this fact to determine how much time has elapse if the following fractions of strontium remain.

a) $\frac{1}{4}$	b) $\frac{1}{8}$	c) $\frac{1}{32}$	d) $\frac{1}{20}$
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Answers 1. a) 5 b) 4 c) 3 d) 3 e) no solution f) 2 g) 4 h) $x=2n+1$, where $n \in \mathbb{I}$ (i.e. x is any odd integer) i) $x=2n$, $n \in \mathbb{I}$ j) -6 k) $\frac{1}{2}$ l) 2 m) 3 n) 0 o) $\frac{2}{7}$ p) 0 q) -2 r) -3 s) 5 t) -3 u) $\frac{2}{3}$ v) 4 w) $\frac{1}{3}$ x) -1 2. a) 5 b) 3 c) -3 d) 2 3. a) -1 b) 1 c) -2 d) 7 e) 4 f) 2, -3 g) 1, 2 h) 4, -3 4. $x=-17, y=2$ 5. a) 3h b) 9h c) 18h d) 7.3h 6. a) 56a b) 84a c) 140a d) 121 years