

4.3B – Transforming $f(x) = b^x$

Complete the following table to investigate various transformations on the exponential functions.

x	$f(x) = 2^x$	$f(x) = 2^x + 1$	$f(x) = 2^x - 3$	$f(x) = 3 \cdot 2^x$	$f(x) = (\frac{1}{2}) \cdot 2^x$	$f(x) = (2)^{x+1}$
-3	$1/8 = 0.125$	1.125	-2.875	0.375		
-2	0.25	1.25	-2.75	0.75		0.5
-1	0.5	1.5	-2.5	1.5	0.25	1
0	1	2	-3	3	0.5	2
1	2	3	-1	6	1	4
2	4	5	1	12	2	8
3	8	9	5	24	4	16

Asymptote is moving from $y=0, y=1, y=-3$ (i.e. translating)

Recall general transformations to any function:

$$g(x) = a f(x - h) + v$$

Translate vertically

Applying this to the exponential functions;

$$g(x) = a b^{(x-h)} + v$$

where $b > 0$

Stretch and reflection by a

Translate horizontally

Example 1: Graphing the following exponential functions.

a) $f(x) = 2^x + 3$

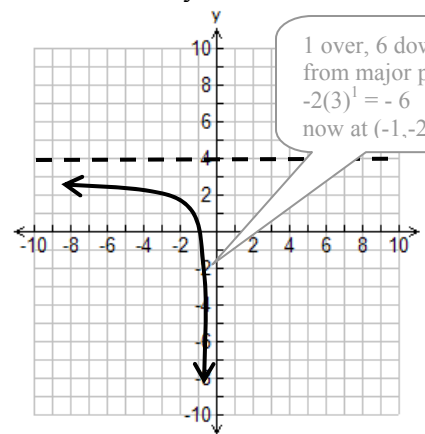
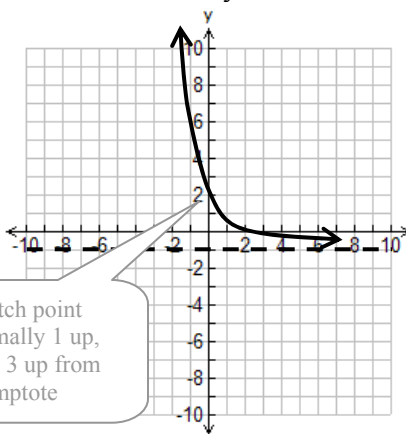
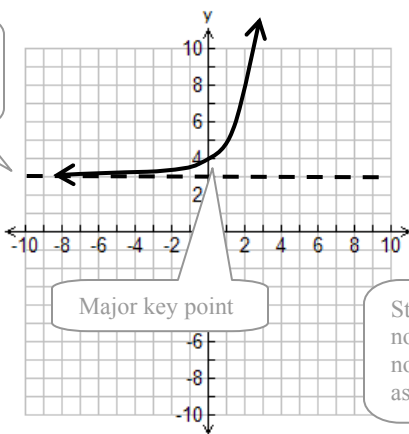
b) $f(x) = 3(\frac{1}{2})^x - 1$

c) $f(x) = (-2)3^{x+2} + 4$

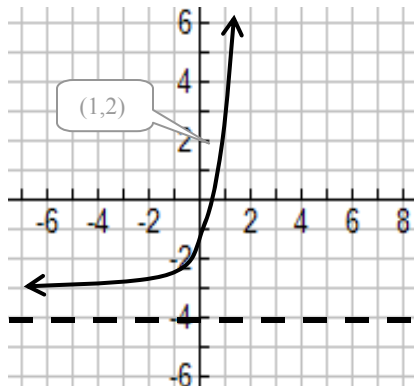
translation H:
V: +3
Stretch H:
V:

trans H:
V: -1
stretch H:
V: by 3

trans H: -2
V: +4
stretch H:
V: by -2



Example 2: Given the function $y = g(x)$ below



a) describe its domain

D: $x \in \mathbb{R}$

b) describe its range

R: $y > 0$

c) determine its equation

know: $y = 3 \cdot b^x - 4$
sub (1,2): $2 = 3b^1 - 4$
 $6 = 3b$
 $2 = b$

Use point where x is 1 so exponent on base is 1 allowing one to solve easily

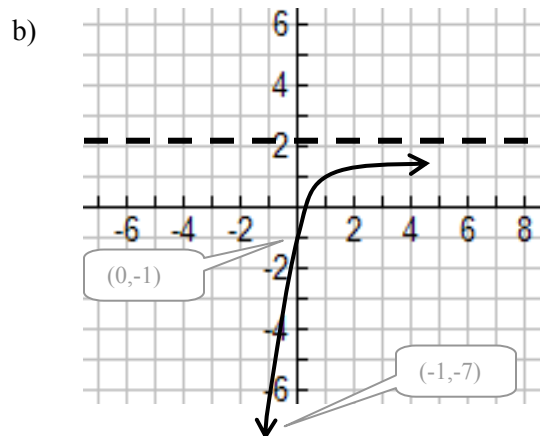
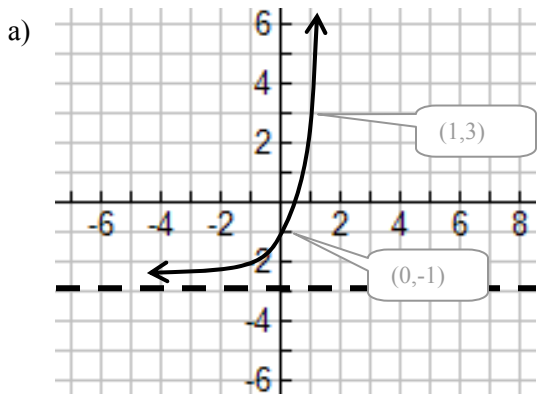
therefore: $y = 3 \cdot 2^x - 4$

Check final answer by applying transformations to see if matches graph

4.3B – Transforming $f(x) = b^x$ Practice Questions

- Sketch the functions $y = 3^x$ and $y = 3^x + 2$ on the same grid
 - What parts of the curve are similar? What parts of the curve are different?
 - Describe the domain and range of each curve
- Sketch the following exponential functions, clearly indicate asymptote, and at least three key points.

a) $f(x) = 3^x - 1$	b) $g(x) = (1/4)^x + 2$	c) $y = (1/2) 5^x$
d) $m(x) = (3)2^x + 3$	e) $y = (-2)3^x$	f) $h(x) = -3(1/4)^x - 1$
g) $y = 4^{x-3} + 2$	h) $y = (-2)^x + 1$	i) $y = 2^{-x} + 1$
j) $f(x) = 5(2/3)^x + 1$	k) $g(x) = 4(1/3)^x$	l) $h(x) = 5^{x+3}$
m) $y = 2^{x-1} - 3$	n) $g(x) = -4(1/3)^{x+2} - 1$	o) $h(x) = 3^{2x-4} + 1$
- How are the functions $y = (8)2^x$ and $y = 2^{x+3}$ related. Show this algebraically.
- The number of insects in a colony can be modeled by the function $P(d) = I_0 (1.4)^d + 5$ where I_0 is initial number in a colony and d is the number of days elapse since starting to count.
 - Is the population increasing or decreasing over time? How do you know?
 - If the initial population is 200, how many insects would be expected after 10 days.
- The worlds population is growing according to the function $P(y)=6(1.046)^{y-2007}$, where $P(y)$ gives population in billions according to calendar year.
 - Why does the exponent have $y - 2007$?
 - Use this model to predict the worlds population in 2025.
- Given the graphs below, determine the exponential function that best models it.



Answers 1. a) both have same shape, one is just a translation of the other, asymptote is on $y=2$ for $y=3^x+2$ **b)** both have $D:x \in \mathbb{R}$ and for $y=3^x$ $R:y > 0$ while for $y=3^x+2$ $R:y > 2$ **2.** check on graphing calculator **3.** They are the same functions, $8=2^3$ which can be re-written like other function using exponent rules **4. a)** increasing because the base is bigger than one (i.e. $b > 1$) and exponent is positive zero **b)** 5790 **5. a)** this is too adjust the year back to equations starting point which must have been 2007 **b)** 13.5B **6. a)** $y=2(3)^x-3$ **b)** $y = -3(1/3)^x+2$

4.3B - Sketching Practice Sheets

