4.4 – Applications of the Exponential Function

Many natural processes that involve growth or decomposition (decay) can be modelled using the exponential functions. Below are some examples.

Example 1: A population of bacteria doubles every 4 hours. Determine an equation to model the bacteria growth as a function of time.

Starting with "x" bacteria

Often easier to think in terms of interger numbers instead of variables

As a function of generation

Generation	Time (h)	# of bacteria	# of bacteria (using actual numbers)	# of bacteria (in exponetial form)	Equation
1	0	x	1	2^0	
2	4	2x	2	2^1	$n = 2^{g-1}$
3	8	4x	4	2^3	or
4	12	8x	8	2^4	$n = 2^{t/4}$
5	16	16x	16	2^5	

As a function of time (hours)

Initially easier to come up with equation using consecutive counting numbers

Therefore exponetal as a function of time would be: $P(t) = x2^{\frac{t}{4}}$ with *t* in hours

Example 2: If human population doubles every 35 years, estimate the World population in 2050 given population in 1999 was about 6B.

know the following: base = 2 (doubling)
time interval = 35
year adjustment = y – 1999

If tripling would use 3
If 8% increase would use 1.08

Time it takes to double

Therfore exponential model: $P(y) = 6 \cdot 2^{\frac{y-1999}{35}}$

Need to adjust exponent so that after 35 years you will have done one doubling (i.e. exponent is 1)

So in 2050: $P(2050) = 6 \cdot 2^{\frac{2050-1999}{35}}$

$= 16.5B$

Example 3: The half life of radium is 1620 years. If have 10mg of radium today, how much will one have in 50 years?

Time it takes for half of subtance to decay

know the following: base = ½ (halfing each time)
original amount = 10
time interval = 1620

This will multiple any effect of your exponent.
So if start with 40 mg then 40 x base
And if start with x mg then x · b

Therfore exponential model: $A(t) = 10 \cdot \left(\frac{1}{2}\right)^{\frac{t}{1620}}$

So in 50 years: $A(50) = 10 \cdot \left(\frac{1}{2}\right)^{\frac{50}{1620}}$

Key into calculator to evaluate in 1 step

$= 9.79mg$

In general: $$A_f = A_o (base)^{\frac{t}{t_i}}$$ where

A_f = final amount at end
A_o = original amount
base = ½ if decay, 2 if doubling, or ? to suit situation
t = elaspsed time

t_i = time interval it takes to double, half-life or ?

4.4 – Applications of the Exponential Function Practice Questions

1. In 1930 a towns population was 125000. In 1992 it was 500000. Estimate population in 2020.

   Know: 500000 = 125000 B ^ (1992 - 1930)
   $4 = B^{62}$
   B = 1.0226

   62nd root of 4.
   Use y^x key and (1/62)

   So: P(y) = 125000 (1.0226) ^ (y – 1930)

   Then: P(2020) = 125000 (1.0226) ^ (2020 – 1930)
   = 935122

   Estimate can be rounded to nearest thousand

   Question in words.
   Answer in words.

   Therefore town population in 2020 should be around 935000

2. Light intensity is reduced by 2.5% for every meter of water in penetrates into. If we start with 100% light at the surface then $I(d) = 100\,(0.975)^d$, where d is depth of water in meters.

   a) Why is the base 0.975 and not 1.025?
   b) What is light intensity at 5m, 10m, 15m, 20m, 25m?
   c) When will light intensity be less than 1%?

3. A bamboo plant increases its height by 12% every day. How tall will the plant be after 4 weeks given it starts at a height of 40 cm?

4. Given that a ball bounces to 70% of its previous height on each successive bounce, come up with an exponential function to model the ball height as a function of the number of bounces, where h_o is initial height dropped from. Use your equations to find the balls height after 7 bounces if dropped from 25 meters.

5. If paid an interest rate of 8% /a compounded annually, come up with an equation to model the total amount as a function of n years. Use an exponential model to determine total amount in your account after 15 years on a $5000 investment.

6. The half life of carbon-14 (C_{14}) is about 5760. If a bone sample only has 30% C_{14} remaining, determine the age of the bone.

7. A helium balloon looses 10% of its volume every 6 hours. What percentage of the original volume will be remaining after 27 hours?

8. A original sample of 160 mg of radioactive sodium (Na_{24}) was reduced to 20 mg after 45h.

   a) Determine half-life of Na_{24}?
   b) Model Na_{24} remaining as function of time.
   c) If you require 100 mg of Na_{24} 12h from now, how much will you need now?

Answers 2. a) because reducing by 2.5 means 97.5=100-2.5 remains **b)** 88%,78%, 68%, 60%, 53% **c)** around 181m **3.** $H(t)=40(1.12)^t$, 9.55m **4.** $H(b)=h_0(0.70)^b$, 2.06m **5.** $A(y)=A_o(1.08)^y$, $15860.85 **6.** $A=100(½)^{t/5760}$, 10004a **7.** V=100(0.90)^(t/6), 62% **8. a)** 15h **b)** A=160(½)^(t/15) **c)** 174mg