

4.5 – Modeling Exponential Data

Continuing on with the theme of using exponential functions to model real life situations, One could analyze a given set of data to see if this might fit best into such an exponential model.

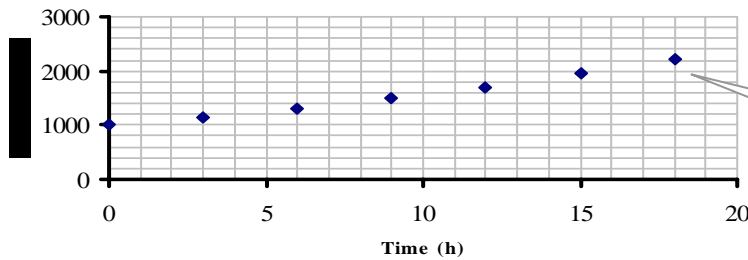
Example 1: The bacteria population in a dish was measured every 3 hours with the data recorded in the table below.

Time (h)	# of bacteria
0	1000
3	1135
6	1307
9	1490
12	1696
15	1957
18	2228

Time (h)	interval	# of bacteria
0	0	1000
3	1	1135
6	2	1307
9	3	1490
12	4	1696
15	5	1957
18	6	2228

- a) Enter the data into a calculator and graph the data.

Figure 1: Bacteria vs. Time



For some scenarios it is useful to adjust original time into intervals to reduce size of numbers. Of course you will need to adjust back in end

Almost looks linear? Compressing vertical exaggerates.

- b) Perform a regression to determine if an exponential model is suitable.

Linear $R^2 = 0.9860$ where: $a = 996.986$
 Quad $R^2 = 0.9998$ $b = 1.0456$
 Expo $R^2 = 0.9999$ so best model is: $P(t) = 997 \cdot 1.0457^t$

Base on data this number should be 1000 but fitting the data to model slightly adjusted this value

Add the function to your plot

- c) Use your model to estimate the bacteria population at 10h, 50h.

$$P(10) = 997 (1.0457)^{10} \approx 1559$$

$$P(50) = 997 (1.0457)^{50} \approx 9268$$

Use two given points to calculate slope of secant.

- d) Calculate the average rate of bacterial growth for the periods;

i) first 6 hours rate = $(1307-1000)/(6-0) = 51$ bacteria/hour
 ii) from 6 to 12 hours rate = $(1696-1307)/(12-6) = 65$ bacteria/hour
 iii) from 12 to 18 hours rate = $(2228-1696)/(18-12) = 89$ bacteria/hour

Rate is increasing as population grows.

- e) Determine the instantaneous rate of growth at the following times.

i) At 0 hours rate = 44 bacteria/hour
 ii) At 9 hours rate = 67 bacteria/hour
 iii) At 18 hours rate = 100 bacteria/hour

Use the trace function and calc (dy/dx) to get slope of tangent on calculator

Draw in tangent on your graph if doing manually

Using the time interval numbers will give you small number to work with but you will have to adjust for years in your exponent to make equation a straight plug in with calendar year

4.5 – Modeling Exponential Data Practice Questions

1. The table below documents the Worlds population over the last 250 years.

Time interval	0	1	2	3	4	5
Year	1750	1800	1850	1900	1950	2000
Population (in billions)	0.79	0.98	1.26	1.65	2.52	6.06

- Enter the data in a graphing calculator and determine the exponential function that best models the data.
- Using your model, predict the World's population in 2050.
- Use your model to predict the year when the World's population reaches 7B.

2. The table below documents the carbon dioxide concentration in the atmosphere.

Year	1860	1880	1900	1920	1940	1960	1980	2000
CO ₂ Concentration (in parts per million)	294	296	300	307	308	319	340	377

- Enter the data in a graphing calculator and determine the exponential model.
- Using your model, to estimate was the CO₂ concentration was in 1930, 1990.
- If the trend continues when will the CO₂ concentration reach 390 parts per million?

3. The table below shows the number of bacteria grown over a 10 hour period in a laboratory. It assumes a zero death rate during this period.

Time	0	1	2	3	4	5	6	7	8	9	10
# of bacteria	1	4	16	64	256	1024	4096	16384	65536	262144	1048576

- Enter the data in a graphing calculator and determine the exponential function model.
- Using your model, to estimate was the bacteria population after 2 ¼ hours.
- If the trend continues what will bacteria population be after 15 hours
- At what time was the bacteria population around 100000?
- Calculate the average rate of growth for the following periods;

- The entire 10 hour period of the experiment
- The last 5 hours of the experiment
- The first 5 hours

- Predict the highest instantaneous rate of growth and when it occurred.
- Determine the instantaneous rate of growth for the following times;

- At 5 hours
- At 7 hours
- At 10 hours

3d is off according to table but it is what equation predicts.

Answers 1. a) $P(y)=1.156 \times 10^{-6}(1.00763)^y$ or $P(y)=0.660(1.462)^{(y-1750)/50}$ **b)** 6.45B **c)** 2061 **2. a)** $C(y)=15.589(1.00156)^y$ or $C(y)=283.843(1.032)^{(y-1860)/20}$ **b)** about 317000 and 348000 **c)** 2062 **3. a)** $P(h)=4^h$ **b)** 23 **c)** about 107B **d)** 8.3h **e) i)** 104857 bacteria/hour **ii)** 209510 b/h **iii)** 205 b/h **f)** The highest instantaneous growth will be at the end of the experiment as population is growing exponential (doubles every ½ hour) **g) i)** 1420 b/h **ii)** 23000 b/h **iii)** 1450000 b/h