4.6 – Logarithms Introduction

We have already seen that a pattern resulting form repeated multiplications generates an exponential pattern that can be models using an exponential function.

Ex. You tell 2 friends a secret, who in turn tell two friend and so on, and so on

2
2 x 2
2 x 2 x 2 → develops pattern $2^0, 2^1, 2^2, 2^3, \ldots 2^x$
to come up with function $f(x) = 2^x$

This function proved useful in many applications. However, one can run into problems when trying to solve for unknowns in the exponent (ex. $23 = 7^x$). As ***logarithms functions*** are the inverse, {i.e. (x,y)→ (y,x)}of exponential functions they can address this problem.

Ex. Given $y = 2^x$ then inverting gives $x = 2^y$
or in function notation $y = \log_2 x$

> Exponential form

> Logarithmic form

We read logarithmic statements as "y" equals "log x" base "2". This is understood as trying to think of what exponent y must one raise base "2" to get "x". In this sense, "x" is the answer.

Ex. $y = \log_{10} 100$ reads as log 100 base 10 *or* just log 100
and means what exponent "y" do you raise 10 to get 100

> Base 10 understood

> As base 10 is the most common, when a base is not written, then 10 assumed (sort of like 2 and the square root sign)

Example 1: Complete the following table

Exponential form	$3^4 = 81$		$5^3 = 125$		$y = 6^2$	
Logarithmic form	$4 = \log_3 81$	$\frac{1}{2} = \log_{25} 5$		$-3 = \log_{\frac{1}{2}} 8$		$y = \log 10$

Example 2: Evaluate the following as exact answer.

> Any variable will do (x,y,?)

a) $\log_5 25$

> Re-write in exponent form, as you are more familiar with

set $y = \log_5 25$
then $5^y = 25$
so $5^y = 5^2$
$y = 2$

b) $\log_{1/3} 27$

so $(1/3)^x = 27$
$3^{-x} = 27$
$3^{-x} = 3^3$
$x = -3$

c) $\log_{\sqrt{3}} 9$

$\left(3^{\frac{1}{2}}\right)^x = 3^2$

> Careful using power rule

$\frac{x}{2} = 2$

$x = 4$

Example 3: Use calculator to evaluate to nearest hundredth.

> Calculators are set up in base 10, so you can just key in

a) $\log_{10} 430$

$= 2.63$

b) $\log 0.0231$

$= -1.64$

> 10 to what exponent gives 0.0231?

c) $\log_3 50$

$=$

> As most calculators use base 10 we cannot do this question without changing the base. As not all calculators allow this, we will be learning the base change property later

Example 4: Solve for unknown.

a) $\log_x 27 = 3$

so $x^3 = 27$ $\therefore x = 3$

> Re-write in exponential form and solve for unknown

b) $3 = \log_5 x$

so $5^3 = x$ $\therefore x = 125$

As logarithms are new and a rather strange concept lots of practice is necessary to get familiar with them

4.6 – Logarithms Introduction Practice Questions

1. Change to logarithmic form

a) $5^2 = 25$ b) $512^{(1/3)} = 8$ c) $144^{-½} = 1/12$ d) $\sqrt{16} = 4$

e) $10^5 = 100\ 000$ f) $27^{-\frac{1}{3}} = \frac{1}{3}$ g) $3^7 = 2187$ h) $8^{\frac{2}{3}} = 4$

i) $7^3 = 343$ j) $16^{1.5} = 64$ k) $8^4 = 4096$ l) $5^{-2} = 0.04$

m) $6^2=36$ n) $9^0=1$ o) $2^{-2}=0.25$ p) $4^4=256$

q) $3^2=9$ r) $2^{3/2}=\sqrt{8}$ s) $3^3=27$ t) $16^{½}=4$

2. Change to exponential form

a) $\log_6 36 = 2$ b) $\log_9 1 = 0$ c) $\log_2 0.25 = -2$ d) $\log_4 256 = 4$

e) $\log_3 9 = 2$ f) $\log_2 \sqrt{8} = 3/2$ g) $\log_3 27 = 3$ h) $\log_{16} 4 = ½$

i) $\log_5 625 = 4$ j) $\log_3 3 = 1$ k) $\log_{10} 1 = 0$ l) $\log_2 8 = 3$

3. Evaluate as exact number

a) $\log_7 49$ b) $\log_5 (1/5)$ c) $\log_2 32$ d) $\log_3 27$

e) $\log_2 8$ f) $\log_{25} 5$ g) $\log_5 1$ h) $\log_6 6$

i) $\log_4 64$ j) $\log_8 \sqrt{8}$ k) $\log_2 8 + \log_3 81$ l) $\log_2 128 - \log_2 32$

m) $\log 100$ n) $\log_4 256 - \log_{10} 100$ o) $\log_2 4$ p) $\log_6 1$

4. Evaluate to 2 decimal places

a) $\log_{10} 100$ b) $\log 5$ c) $\log_{10} 275$ d) $\log\ 0.05$

e) $\log\ 4000$ f) $\log_{10} -5$ g) $\log_3 100$ h) $\log_7 100$

5. Solve for x.

a) $2 = \log_4 x$ b) $\log_2 x = 9$ c) $\log_x 16 = 2$

d) $\log_x (1/9) = 2$ e) $\log_3 x = -4$ f) $x = \log_{\sqrt{2}} 32$

Answers 1. a) $\log_5 25=2$ **b)** $\log_{512} 8=1/3$ **c)** $\log_{144}(1/12)=-½$ **d)** $\log_{16} 4=0.5$ **e)** $\log_{10} 100000=5$ **f)** $\log_{27}(1/3)=-1/3$ **g)** $\log_3 2187=7$ **h)** $\log_8 4=2/3$ **i)** $\log_7 343=3$ **j)** $\log_{16} 64=3/2$ **k)** $\log_8 4096=4$ **l)** $\log_5 0.04= -2$ **m)** $\log_6 36=2$ **n)** $\log_9 1=0$ **o)** $\log_2 0.25=-2$ **p)** $\log_4 256=4$ **q)** $\log_3 9=2$ **r)** $\log_2 \sqrt{8}=3/2$ **s)** $\log_3 27=3$ **t)** $\log_{16} 4=½$ **2. a)** $6^2=36$ **b)** $9^0=1$ **c)** $2^{-2}=0.25$ **d)** $4^4=256$ **e)** $3^2=9$ **f)** $2^{3/2}=\sqrt{8}$ **g)** $3^3=27$ **h)** $16^{½}=4$ **i)** $5^4=625$ **j)** $3^1=3$ **k)** $10^0=1$ **l)** $2^3=8$ **3. a)** 2 **b)** -1 **c)** 5 **d)** 3 **e)** 3 **f)** ½ **g)** 0 **h)** 1 **i)** 3 **j)** ½ **k)** 7 **l)** 2 **m)** 2 **n)** 2 **o)** 2 **p)** 0 **4. a)** 2 **b)** 0.70 **c)** 2.44 **d)** -1.30 **e)** 3.60 **f)** no solution **g)** wrtie in exponential form and estimate at 4.19 **h)** 2.37 **5. a)** 16 **b)** 512 **c)** 4 **d)** 1/3 **e)** 1/81 **f)** 10