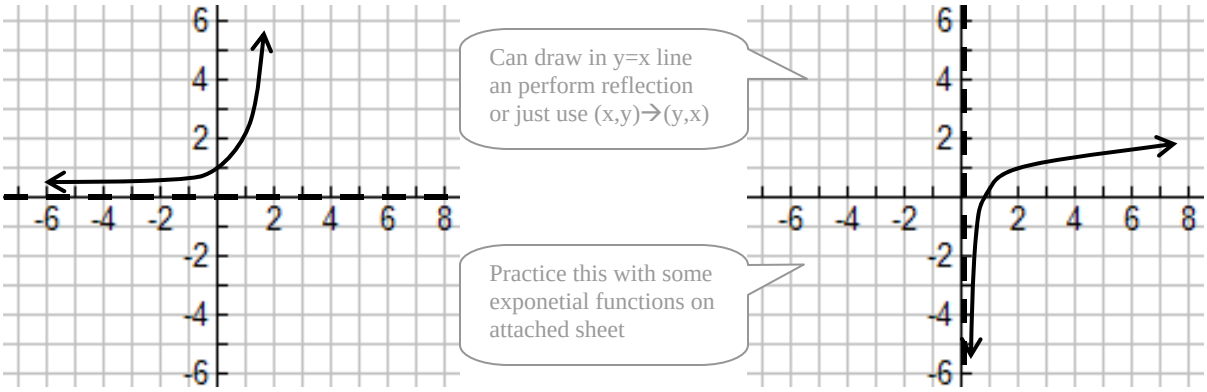
4.7 – Investigating Logarithmic Functions - $f(x) = a\log_b x + c$

Recall that the logarithm function is the inverse of an exponential function. So one could transform an exponential function by switching (i.e. inverse) x & y values to get the corresponding logarithmic function.

Ex. $f(x) = 2^x$ corresponds to $g(x) = \log_2 x$



Use your graphing calculator to sketch each of the following on the same grid and comment on similarities and differences. What point(s) are invariant? How do the other points change relative to each other?

Using the calculator for Set 2 proves difficult as not in base 10? We need to transform!

Change line style so you can tell difference between functions

Set 1	$y_1 = \log_{10} x + 2$	$y_2 = \log(x + 2)$	$y_3 = 2\log x$	$y_4 = \log 2x$
Set 2	$y_1 = \log_2 x$	$y_2 = \log_3 x$	$y_3 = \log_4 x$	$y_4 = \log_{½} x$

Applying transformation to the logarithmic functions; $g(x) = \boldsymbol{a}\ log_B(x - \boldsymbol{h}) + \boldsymbol{v}$ where b>0 & b≠1

Stretch and reflection by a

Translate horizontally

Translate vertically

Example 1: Graph the following logarithmic functions;

a) $f(x) = 3\log_2 x + 1$

base: 2
translation H:
V: +1
Stretch H:
V: by 3

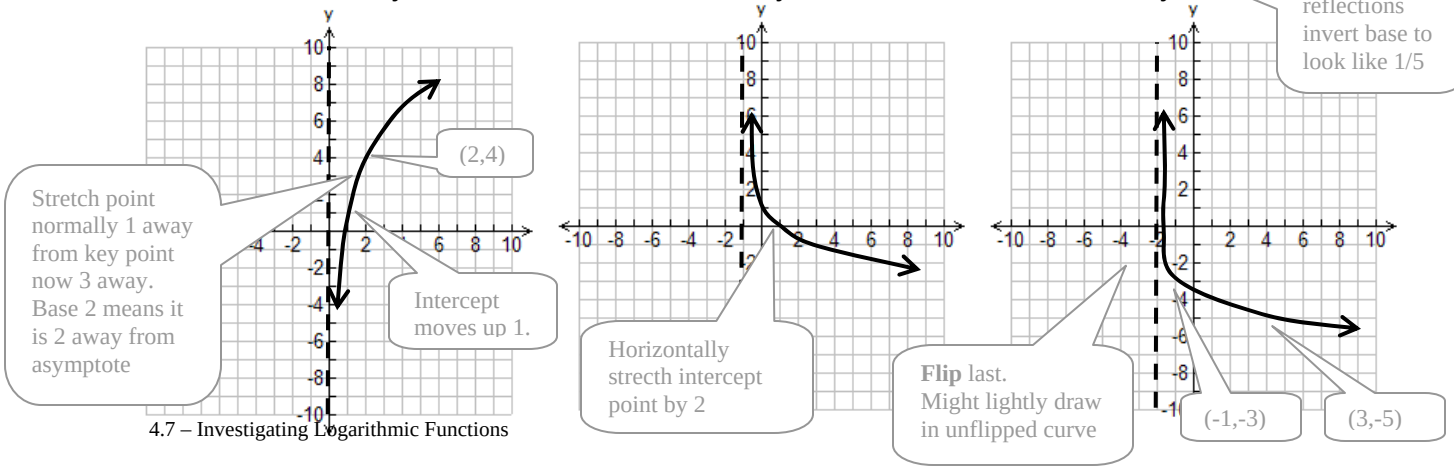
b) $f(x) = 2\log_{½}(½(x + 1))$

base: ½
trans H: -1
V:
stretch H: by 2
V: by 2

c) $f(x) = -2\log_5(x+2) - 3$

base: 5
trans H: -2
V: -3
stretch H:
V: by -2

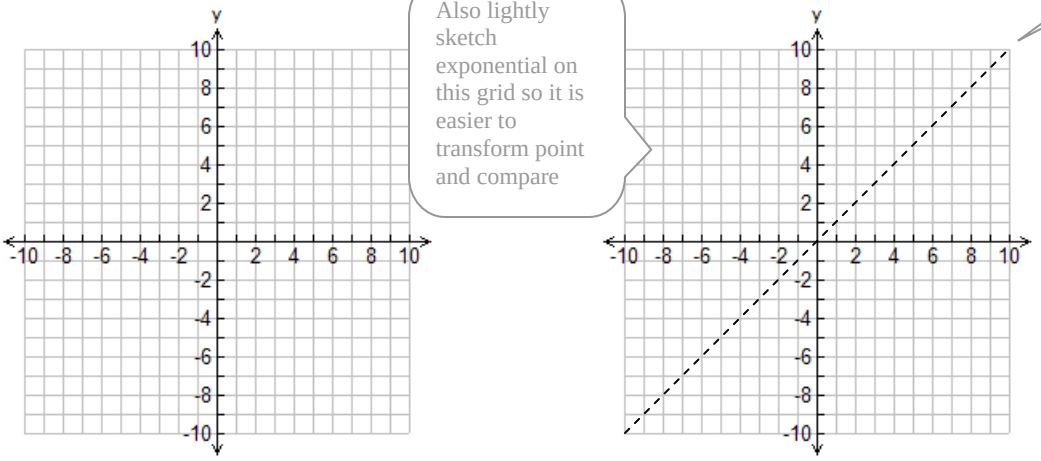
Vertical reflections invert base to look like 1/5



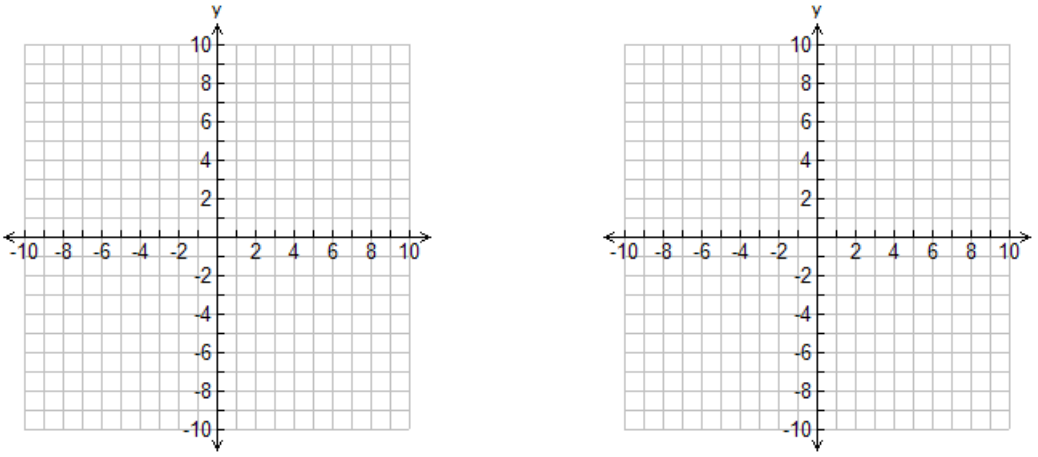
4.7 – Investigating Logarithmic Functions by Inverting the Corresponding Exponential

For each of the following sketch the exponential function first and then invert (x,y)→ (y,x) its' major and key points to get the corresponding logaritmic function.

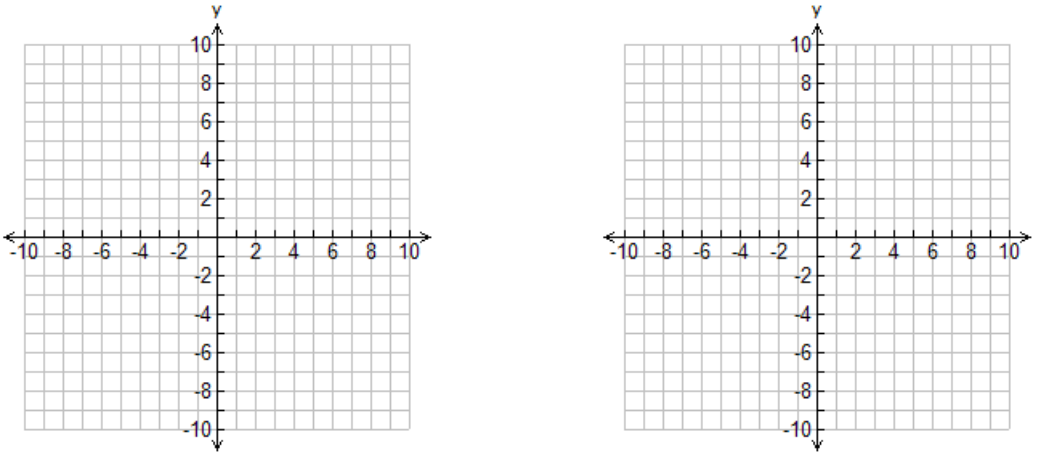
1. $f(x) = 3^x$ $g(x) = \log_3 x$



2. $f(x) = 5^x$ $g(x) = \log_5 x$



3. $f(x) = (½)^x$ $g(x) = \log_{½} x$



4.7 – Investigating Logarithmic Functions Practice Questions

1. Sketch the functions $y = \log_4 x$ and $\log_{¼} x$ on the same grid

   a) What parts of the curve are similar? What parts of the curve are different?
   b) Describe the domain and range of each curve

2. Sketch the following exponential functions; clearly indicate asymptote, and at least two key points.

Use attached grids to sketch questions p to y

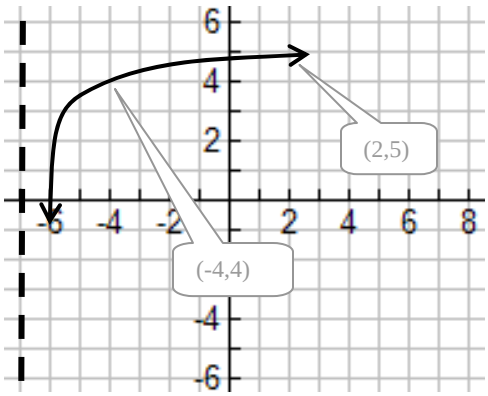
a) $f(x) = \log_{10} x + 1$	b) $f(x) = \log_{10} x - 1$	c) $f(x) = \log_{10} x + 3$
d) $m(x) = \log_{10}(x + 3)$	e) $m(x) = \log_{10}(x - 1)$	f) $m(x) = \log_{10}(x + 3) - 2$
g) $y = 3\log_{10} x$	h) $y = \log_{10} 3x$	i) $y = -3\log_{10} x$
j) $h(x) = \log_{10}(-3x)$	k) $h(x) = \log_{10}(½x)$	l) $h(x) = ½ \log_{10}(x)$
m) $f(x) = \log_{0.1} x$	n) $f(x) = \log_{0.1} x + 3$	o) $f(x) = \log_1 x$
p) $y = \log_2 x$	q) $y = \log_5 x$	r) $y = \log_{1/2} x$
s) $y = -2\log_3(x-1) + 2$	t) $y = 3\log_4(x+2) + 1$	u) $y = -½\log_3(x+1) - 2$
w) $y = -\log_2(½(x+1)) - 3$	x) $y = \log_{0.5}(x-2)$	y) $y = \log_3(-2x+4) + 1$

3. Accurately sketch the logarithmic function $g(x) = 3\log_3(0.2(x+5)) – 2$

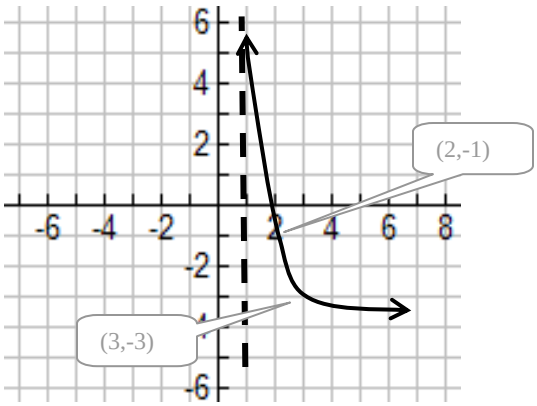
   a) Describe the domain and range
   b) Describe increase/decreasing interval(s)
   c) Determine average rate of change from -4 to 10
   d) Determine instantaneous rate of change i) at -4 ii) at 10

4. Given the graphs below, determine the logarithmic function that best models it.

a)

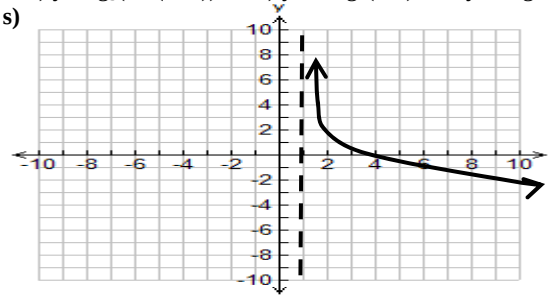


b)

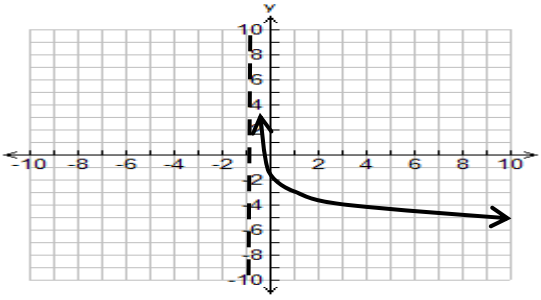


Answers 1. a) both have asymptote of x=0 and x-intercept of x=1, ¼ key point becomes 4 key point vertically reflected, when b>0 (i.e. $\log_4$)function increase, when 0<b<1(i.e. $\log_{¼}$) function decreases **b)** both have D:x>0 and R:y∈R **2.** check base 10 graphs on calculator, some of the other ones are shown below
3. a) D:x>-5 and R:y∈R **b)** g(x) increases when x>-5 **c)** about 1/2 **d)** about 8/3 and 1/10
4. a) $y=\log_3(1/3(x+7))+4$ **b)** $y=-2\log_2(x-1)-1$ or $y=2\log_{½}(x-1)-1$

s)



w)



4.7 - Sketching Practice Sheets