4.8A – Properties of Logarithms

Through various rearrangements in exponential form we can come up with the following basic properties to facilitate our evaluation of logarithms.

a) $\log_b 1 = 0$ ex. $\log_5 1 = 0$ — What exponent do you have to put base '5" to get 1?

b) $\log_b b = 1$ ex. $\log_5 5 = 1$ — What exponent do you have to put base '5" to get 5?

c) $\log_b b^x = x$ ex. $\log_5 5^x = x$ — This follows from previous example. That is exponent needed to get 5^x?

d) $b^{\log_b x} = x$ ex. $b^{\log_b x} = x$

In addition to the basic properties the following operational properties prove useful when trying to rearrange logarithmic expressions.

See investigation sheets to see how these properites might have developed

a) $\log_a x^r = r \log_a x$ ex. $\log_2 5^3 = 3 \log_2 5$ — Exponent moves in front to become multiplier

b) $\log_a xw = \log_a x + \log_a w$ ex. $\log_5 (3)(4) = \log_5 3 + \log_5 4$

c) $\log_a\left(\frac{x}{w}\right) = \log_a x - \log_a w$ ex. $\log_5\left(\frac{2}{3}\right) = \log_5 2 - \log_5 3$

Example 1: Express as single logarithm

a) $\log_5 8 + \log_5 30$

$= \log_5 240$ — 240=8x3

b) $\log 150 - \log 25$

$= \log 6$ — 6=150/25

c) $3\log(x+3) - 2\log(x-1)$

$= \log(x+3)^3 - \log(x-1)^2$

$= \log\frac{(x+3)^3}{(x-1)^2}$

Example 2: Simplify the following

a) $\log_4 16$

$= \log_4 4^2$

$= 2$

b) $\log_3\left(\frac{27}{81}\right)$

$= \log_3 27 - \log_3 81$

$= \log_3 3^3 - \log_3 3^4$

$= 3 - 4$

$= -1$

c) $\log_2 48 - \log_2 3$

$= \log_2\left(\frac{48}{3}\right)$

$= \log_2 16$

$= \log_2 2^4$

$= 4$

d) $3^{-\frac{1}{2}\log_3 49}$

$= 3^{\log_3 49^{-\frac{1}{2}}}$

$= 3^{\log_3\left(\frac{1}{7}\right)}$

$= \frac{1}{7}$

Example 3: Solve the following

a) $7^x = 400$

$\log 7^x = \log 400$

$x \log 7 = \log 400$

$x = \log 400 \div \log 7$

$x = 3.08$

Take **log** base 10 of both sides. This is similar to squaring both sides, it is just a different operation (or function).

base 10 now allow you to use calculator

b) $7(1.06)^x = 5.20$

$\log 7(1.06)^x = \log 5.2$

$\log 7 + x \log 1.06 = \log 5.2$

$x \log 1.06 = \log 5.2 - \log 7$

$x = (\log 5.2 - \log 7) \div \log 1.06$

$x = -5.10$

Written in one line this looks like.

$$x = \frac{\log 5.2 - \log 7}{\log 1.06}$$

c) $\log_3 50$

We now have a way or rewriting this to solve. One can memorize the rearrangement to come up with base change technique

set $3^x = 50$

$x \log 3 = \log 50$

$x = \log 50 \div \log 3$

$x = 3.56$

$$x = \frac{\log 50}{\log 3}$$

4.8A – Properties of Logarithms Investigation Sheet

Investigation A

Use a calculator to evaluate the following logarithms (all to base 10):

Log 4 = ____________________
Log 40 = ____________________
Log 400 = ____________________
Log 4000 = ____________________

What would you expect the answer to be for log 40 000? ______________

How did you arrive at that answer?

Use a calculator to evaluate the following common logarithms:

Log 1 = ____________________
Log 10 = ____________________
Log 100 = ____________________
Log 1000 = ____________________

If you rewrite log 40 as log (10 x 4), how would you rewrite its numerical value, so as to take into account the product (10 x 4)?

Rewrite the following:

Log 400 =
= ____________________
Log 4000 =
= ____________________

Write a generalization for the pattern:

Given the values of log 24 and log 4, how could you find the value of log 96?

How could we rewrite the general case, $\log_a(xy)$ **?**

4.8A – Properties of Logarithms Investigation Sheet

Investigation B

Use a calculator to evaluate the following logarithms (all to base 10):

Log 40 000 = ____________________
Log 4000 = ____________________
Log 400 = ____________________
Log 40 = ____________________

What would you expect the answer to be for log 4? ______________

How did you arrive at that answer?

Use a calculator to evaluate the following common logarithms:

Log 1000 = ____________________
Log 100 = ____________________
Log 10 = ____________________
Log 1 = ____________________

If you rewrite log 4 as log (40 ÷ 10), how would you rewrite its numerical value, so as to take into account the quotient (40 ÷ 10)?

Rewrite the following:

Log 4000 =
= ____________________
Log 400 =
= ____________________

Write a generalization for the pattern:

Given the values for log 96 and log 4, how could you find the value of log 24?

How could we rewrite the general case, $\log_a\left(\frac{x}{y}\right)$ **?**

4.8A – Properties of Logarithms Practice Questions

1. Express as single logarithm and simplify

a) $\log_5 15 - \log_5 3$
b) $\log_2 48 - \log_2 6$
c) $\log_6 9 + \log_6 4$
d) $\log_6 4 + \log_6 3 + \log_6 3$
e) $\log_3 45 - \log_3 5 + \log_3 3$
f) $\log_8 48 + \log_8 4 - \log_8 3$
g) $3\log 10 + 2\log 100$
h) $2\log 5 + 3\log 2 - \log 2$
i) $\log_3 3 + \log_5 1$

2. Express as sum of logs

a) $\log_3 (4)(5)$
b) $\log 50$
c) $\log_2 28$
d) $\log_6 54$

3. express as difference of logs

a) $\log (5/15)$
b) $\log_{11} 21$
c) $\log_2 13$
d) $\log_3 12$

4. Simplify and evaluate

a) $\log_3 12 - \log_3 4$
b) $2\log 5 + \log 4$
c) $\log_4 192 - \log_4 3$
d) $\log_3 9^3$
e) $\log_5 125^4$
f) $\log_2 16 - \log_2 8$
g) $\log_2 20 - \log_2 5$
h) $\log_2 6.4 + \log_2 10 - \log_2 8$
i) $2\log 9 + \log 9 – \log 3$
j) $(\log_2 100) \div (\log_2 10)$
k) $\dfrac{\log_{10} 27}{\log_{10} 3}$
l) $\dfrac{\log_2 16}{\log_2 8}$
m) $\log_{12} 4 + \log_{12} 36$
n) $\log_{10} 3 + \log_{10} (1/30)$
o) $10^{-3\log_{10} 2}$

5. Solve to 2 decimal places.

a) $10^{2x} = 496$
b) $5^{3x+4} = 25$
c) $3^{5-2x} = 875$
d) $7^{x+9} = 56$
e) $5^x \cdot 3^{2x} = 92$
f) $2 \log x = \log 32 + \log 2$

6. Use your graphing calculator to solve the following to 2 decimal places.

a) $5^x + 3^{2x} = 92$
b) $\log_3 x - \log_x 7 = 100$

Answers 1. a) $\log_5(15/3)=1$ **b)** $\log_2(48/6)=3$ **c)** $\log_6(9\cdot4)=2$ **d)** $\log_6(4\cdot3\cdot3)=2$ **e)** $\log_3((45/3)\cdot3)=3$ **f)** $\log_8((24\cdot3)/3)=2$ **g)** $\log(10^3\cdot100^2)=7$ **h)** $\log((5^2\cdot2^3)/2)=2$ **i)** different bases do not allow one to express as single log **2.** answer many vary from **a)** $\log_3 4+\log_3 5$ **b)** $\log 5+\log 10$ **c)** $\log_2 4+\log_2 7$ **d)** $\log_6 6+\log_6 9$ **3.** answer may vary from **a)** $\log 5-\log 15$ **b)** $\log_{11} 42-\log_{11} 2$ **c)** $\log_2 26-\log_2 2$ **d)** $\log_3 36-\log_3 3$ **4. a)** 1 **b)** 2 **c)** 3 **d)** 6 **e)** 12 **f)** 1 **g)** 2 **h)** 8 **i)** 2.38 **j)** 2 **k)** 3 **l)** 4/3 **m)** 2 **n)** -1 **o)** 1/8 **5. a)** 1.35 **b)** -2/3 **c)** -0.58 **d)** -6.93 **e)** 1.19 **f)** ±8 **6. a)** 1.93 **f)** no solution