4.9 – Change of Base with Logarithms

In the last section we recognized that it is possible to evaluate logarithms with bases other than 10 by using a combination of re-writing in exponential form, taking base 10 logarithm (s0 we can use calculators) of both sides, and applying logarithmic properties to rearrange and solve.

Ex. given $\log_3 50$
set $x = \log_3 50$
so $3^x = 50$
$x \log 3 = \log 50$
$x = \log 50 \div \log 3$
$x = 3.56$

Take **log** base 10 of both sides. But really we could take log of any base we choose

From this we can come up with a general form when needing to change the base of the logarithm. Realize that in the vast majority of cases we will be changing to base 10 so that we can use the calculator to evaluate, but really any base can be used.

In general: $\log_b x = \dfrac{\log_a x}{\log_a b}$ and also $\log_b x = \dfrac{1}{\log_x b}$

Example 1: Evaluate

a) $\log_3 23$

$= \dfrac{\log 23}{\log 3}$

$= 2.85$

Use calculator

b) $\log_5\left(\dfrac{1}{2}\right)$

$= \dfrac{\log 0.5}{\log 5}$

$= -0.43$

Do not confuse dividing one log by another with the subtraction property which is the division of numbers within one logarithm

Example 2: Re-write so that can graph on calculator

a) $y = \log_5 x$

$y = \dfrac{\log x}{\log 5}$

$y = \dfrac{1}{\log 5} \cdot \log x$

$y = (1.43)\log x$

This part is just a number like any (1/?)

b) $y = \log_{1/3} x$

$y = \dfrac{\log x}{\log 0.33}$

$y = -2.09 \log x$

If you forget the base change conventions then just go back to the basic and write out the long way

Example 3: Show the following $\dfrac{1}{\log_3 a} + \dfrac{1}{\log_4 a} = \dfrac{1}{\log_{12} a}$

Show is not as formal as a proof, so format is up to you.

$LS = \log_a 3 + \log_a 4$

$= \log_a 12$

$= \dfrac{1}{\log_{12} a}$

4.9 – Change of Base with Logarithms

1. Re-write using base 10.

a) $\log_4 20$	b) $\log_6 0.25$	c) $\log_2 10$	d) $\log_8 (1/3)$
e) $\log_7 15$	f) $\log_x 12$	g) $\log_5 0.5$	h) $\log_9 \sqrt{6}$

2. Evaluate to 3 decimal places

a) $\log_4 20$	b) $\log_6 0.25$	c) $\log_2 10$	d) $\log_8 (1/3)$
e) $\log_7 15$	f) $\log_2 12$	g) $\log_5 0.5$	h) $\log_9 \sqrt{6}$
i) $\log_3 10$	j) $\log_6 0.20$	k) $\log_5 250$	l) $\log_8 0.03$
m) $3\log 100$	n) $2\log_3 10$	o) $6\log_5 10 - 4\log_2 10$	p) $5\log_7 21 + 2\log_3 45$

3. Solve for x, to 2 decimal places

a) $6^x = 55$	b) $13^x = 27$	c) $4^x = 512$	d) $2^x = 0.125$
e) $7^x = 125$	f) $5^{2x} = 39$	g) $4^{3x} = 43$	h) $12^{2x-3} = 144$
i) $0.6^{4x} = 0.734$	j) $12^x = (4)(8^{2x})$	k) $7(0.43)^{2x} = (9)(6^{-x})$	

Answers 1. a) log20/log4 **b)** log0.25/log6 **c)** log10/log2 **d)** log8/log(1/3) **e)** log15/log7 **f)** log12/logx **g)** log0.5/log5 **h)** log√6/log9 **2. a)** 2.16 **b)** -0.77 **c)** 3.32 **d)** -0.52 **e)** 1.39 **f)** 3.58 **g)** -0.43 **h)** 0.41 **i)** 2.10 **j)** -0.90 **k)** 3.43 **l)** -1.69 **m)** 6 **n)** 4.19 **o)** -4.70 **p)** 14.75 **3. a)** 2.24 **b)** 1.29 **c)** 4.5 **d)** -3 **e)** 2.48 **f)** 1.14 **g)** 0.90 **h)** 2.5 **i)** 0.15 **j)** -0.83 **k)** 2.4