4.10 – Solving Logarithmic Equations

We can apply the many properties we have recently learned to solving both logarithmic and exponential equations.

Recall the basics $y = \log_b x$ can be written $x = b^y$ where $b > 0, b \neq 1$

Another strange consideration also pops up when solve some equations.

Ex. Considering $\boldsymbol{y = \log_2 -2}$ we get $\boldsymbol{-2 = 2^y}$ which makes no sense as base differs.
So also need x > 0 for most solutions (can have x < 0 if horizontal translation)

Example 1: Solve the following

a) $\log_6 x = 2$

$6^2 = x$
$36 = x$

b) $3^x = 23$

$\log 3^x = \log 23$
$x \log 3 = \log 23$
$x = \log 23 \div \log 3$
$x = 2.85$

c) $\log_6 x + \log_6 (x + 1) = 1$

$\log_6 x(x + 1) = 1$
$6^1 = x(x + 1)$
$6 = x^2 + x$
$0 = x^2 + x - 6$
$0 = (x + 3)(x - 2)$

$\therefore x + 3 = 0$ or $x - 2 = 0$
$x = -3$ $x = 2$

Checking in original equation we see that substituting -3 would create a negative base situation. Therefore x=-3 is **inadmissible**

$\therefore x = 2$ is only solution

d) $\log \sqrt{y} = \log 1 - 2\log 3$

$\log \sqrt{y} = \log 1 - \log 3^2$
$\log \sqrt{y} = \log (1 \div 3^2)$
$\sqrt{y} = (1 \div 3^2)$
$\sqrt{y} = (1/9)$
$y = (1/9)^2$
$y = 1/81$

Take inverse log of both sides. Similar to undoing square root operation on both sides which we do in a later step by squaring both sides

e) $\log_2(a+2) = 3 - \log_2 a$

$\log_2(a+2) + \log_2 a = 3$
$\log_2 a(a+2) = 3$
$a^2 + 2a = 2^3$
$a^2 + 2a - 8 = 0$
$(a + 4)(a - 2) = 0$

$\therefore x + 4 = 0$ or $x - 2 = 0$
$x = -4$ $x = 2$

Checking in original equation we see that substituting -4 would create a negative base situation. Therefore x=-4 is **inadmissible**

$\therefore x = 2$ is only solution

f) $\log_2 (\log_3 x) = 2$

$2^2 = \log_3 x$
$4 = \log_3 x$
$3^4 = a$
$81 = a$

4.10 – Solving Logarithmic Equations Practice Questions

1. Solve the following:

a) $\log_{10} x = 2$
b) $\log_3 x = -2$
c) $\log_{10} x = 2 \log_{10} 5$
d) $\log_2 2x = 0$
e) $\log_5 10^x = 7$
f) $\log_6 x = 2$
g) $\log_{x^2} 81 = 1$
h) $\log_2 8x = 3 \log_2 x$
i) $\log_3 x = 4 \log_3 3$
j) $\log x = 2 \log 3 + 2 \log 2$
k) $\log \sqrt{x} = \log 1 - 2 \log 3$
l) $2^x - 1 = 4$
m) $\log_6 (x-1) + \log_6 (x+4) = 2$
n) $\log_2 (2x+2) - \log_2 (x-1) = 3$
o) $3 (2^x) = 18^{x-1}$
p) $8^{3x+1} = 64^{2x}$
q) $\log_2 (x+2) = 3 - \log_2 x$
r) $-5^x = 25$
s) $\log_2 x + \log_2 (x+2) = 3$
t) $\log_5 x + \log_{10} x = 5$

2. It is said that a car depreciates 15% per year. Using this rate, how long will it be before a new car is worth half of its original value?

3. Hospitals using radioactive cobalt-60 in some of their medical treatments. When the radioactivity of the cobalt decreases to 45% they need to replace the cobalt with a new one. Given the half-life of Co^{60} is 5.24a, how often do they need to replace it?

4. In 1950 a towns' population was 40000. In 2007 it was 80000. Estimate what the population was in 1980.

5. The half life of carbon 14 (C_{14}) is about 5760. If a bone sample only has 5% C_{14} remaining, determine the age of the bone.

6. A 1950 silver dollar is a highly collectable coin. It was seen listed on e-bay for $500 last week. Calculate the annual rate at which this coin has appreciated.

7. A pot of boiling (100°C) water is removed from the stove to cool. Every 5 minutes the difference between the water and room temperature is reduced by 50%.

   a) Given the room temperature is 20°C, express water temperature as a function of the time, in minutes, since the pot was removed from the stove.
   b) How long will it take the water to cool to 30°C?

Answers 1. a) 100 **b)** 1/9 **c)** 25 **d)** ½ **e)** $7/(\log_5 10)$= 4.9 **f)** 36 **g)** ±9 **h)** $2\sqrt{2}$ **i)** 81 **j)** 36 **k)** 1/81 **l)** 2.32 **m)** 5 **n)** 10/6 **o)** 1.82 **p)** 1 **q)** 2 **r)** no solution **s)** 2 **t)** 114.04 **2.** 4.25a or 4a & 3m **3.** 6a **4.** about 57600 **5.** about 24894 years old **6.** ~11.5% **7. a)** T(t)=80(0.5)^(t/5)+20 **b)** 15 minutes