4.11 – Uses of Logarithms

As our number system is built upon the base 10, the common logarithm provides a useful tool to reduce large numbers into simple integer exponents. John Napier originally introduced (invented) logarithms to serve this purpose.

Standard notation	1	10	100	1000	10000
Exponential notation	10^0	10^1	10^2	10^3	10^4
Logarithmic notation	0	1	2	3	4

Today it allows us to express scales that cover several orders of magnitude in simple terms.

The Richter Scale

The Richter scale was developed in 1935 by seismologist Charles F. Richter. It measures the magnitude of an earthquake by comparing the intensity of the earthquake to some reference earthquake. The formula developed by Richter is;

$$M = \log\left(\frac{I}{I_o}\right)$$

where M is the Richter value used to measure the magnitude of quake

I is the intensity of the earthquake under study
I_0 is the intensity of a reference earthquake

Example 1: Calculate the magnitude of an earthquake that is 1500 times more intense than a reference earthquake.

Given: I_0 = reference
$I = 1500\ I_0$

then:
$$M = \log\left(\frac{1500I_o}{I_o}\right)$$
$$M = \log 1500$$
$$M = 3.2$$

Therefore this earthquake would register a magnitude of 3.2 on the Richter scale.

Example 2: How much more intense is a quake registering 8.1 than one registering 5.4?

Intensity for 8.1 quake (I_a)

$$8.1 = \log_{10}\left(\frac{I_a}{I_o}\right)$$
$$10^{8.1} = \frac{I_a}{I_o}$$
$$10^{8.1} I_o = I_a$$

Assign intensity as I_a so that we can compare to another intensity I_b

Intensity for 5.4 quake (I_b)

$$5.4 = \log_{10}\left(\frac{I_b}{I_o}\right)$$
$$10^{5.4} = \frac{I_b}{I_o}$$
$$10^{5.4} I_o = I_b$$

so comparing I_a to I_b we see:
$$\frac{I_a}{I_b} = \frac{10^{8.1}}{10^{5.4}} = 501$$

Therefore a quake regisiter 8.1 is 501 times more intense than one of 5.4

The Decible Scale

The loudness of any sound is measured relative to the loudness of sound at the threshold of hearing. Sounds at this level are the softest that can still be heard.
The formula used to compare sounds is;

$$L = 10\log\left(\frac{I}{I_o}\right)$$

where L is the loudness measured in decibels ($^1/_{10}$ of a bel).

I is the intensity of the sound being measured
I_0 is the intensity of a sound at the threshold of hearing

At the threshold of hearing, the loudness of sound is zero decibels (0 dB).

Example 3: If a sound is 8000 times more intense that a sound you can just hear (i.e. threshold), calculate its' measure in decibles.

Given: I_0 = threshold
$I = 8000\ I_0$

then:
$$L = 10\log\left(\frac{8000I_o}{I_o}\right)$$
$$L = 10\log 8000$$
$$L = 39dB$$

Therefore this sound would measure 39 dB.

The pH Scale

The pH scale allows chemists to determine the concentration of hydrogen ion in a liquid. It ranges from values of 1 to 14. The higher the pH, the more basic, or less acidic the liquid. The lower the pH, the more acidic or less basic the liquid.

- A liquid with a pH of less than 7.0 is considered *acidic*
- A liquid with a pH of greater than 7.0 is considered *basic*
- A liquid with pH = 7.0 is considered to be *neutral*. Pure water has a pH of 7.0.

The relationship between pH and H^+ ion concentration is inversely proportional so;

- Low pH = High H^+ ion concentration while High pH = Low H^+ ion concentration

The relationship between pH and hydrogen ion concentration is given by the formula;

$$pH = -\log\left[H^+\right]$$

where $[H^+]$ is the concentration of hydrogen ion in moles per litre.

Example 4: Calculate the pH given $[H^+]$ is 10^{-6} moles/litre.

$$pH = -\log\left[10^{-6}\right]$$
$$= -(-6)$$
$$= 6$$

Therefore this solution would have a pH of 6.

4.11 – Uses of Logarithms Practice Questions

1. Calculate the hydrogen ion concentration of a fruit juice of pH 3.10

$$\begin{aligned} pH &= -\log[H^+] \\ 3.10 &= -\log[H^+] \\ -3.10 &= \log_{10}[H^+] \\ 10^{-3.10} &= [H^+] \\ 0.00079 &= [H^+] \end{aligned}$$

Therefore $[H^+]$ is 7.9×10^{-4} moles per litre

2. The formula t = -23 log (T – RT) + 33 gives the elapsed time since death, in hours, where RT is the room temperature and T is the body's measured temperature.
   a) Why is the formula not written using function notation?
   b) Calculate the time of death given the police found a body at 3:00 pm in a 22°C room with a temperature of 32°C.
   c) Describe transformations occurring on the formula and use your knowledge to draw a rough sketch of the graph
3. The San Francisco earthquake of 1989 measured 6.9 on the Richter scale. The Alaska earthquake of 1964 measured 8.5.
   a) How many times as intense as the San Francisco earthquake was the Alaska earthquake?
   b) Calculate the magnitude of an earthquake that is twice as intense as the 1989 San Francisco earthquake.
4. Find the pH of a liquid with an $[H^+]$ of 8.7×10^{-6} mol/L
5. A jet engine sound is 1 000 000 000 000 times more intense than a sound you can just hear. What is the measure of its loudness in decibels? (Wow! When is the last time you used a trillion?)
6. The Ph of water in a small lake has dropped from 5.4 to 4.8 over the last three years. How many more times as acidic is the lake now compared to 3 years ago?
7. Loudness level of a heavy snore is 69 dB. The loudness level of a conversation is 60 dB. The loudness level of a whisper is 30 dB. Compare how many times more intense a snore is than a conversation and then a whisper.
8. Find the $[H^+]$ of milk which has a pH of 6.50.
9. John decided to change the stock muffler on his car to a new high performance free flow style. The orignal muffler had a sound level measured at 75dB while the new muffler will be 120dB. How many times louder is the new muffler?
10. Why does the formula for pH have a negative sign in front of the log?

Answers 2. a) elapsed time depends on more than one factor (i.e. time is a function of room temperature and body temperature) **b)** 10h elapsed time so 5:00am **c)** vertical translation of +33 and vertical stretch of -23. Given (T-RT) can vary depending on body and room temp this means x might go from 0 to 40? See sketch below **3. a)** 40 **b)** 7.2 **4.** 5.06 **5.** 120dB **6.** 4 times more acidic **7.** 7.9 times louder than conversation and 7900 times louder than a whisper **8.** 3.2×10^{-7} mol/L **9.** It will be 32000 times louder **10.** because hydrogen ion concentration is a decimal number this translates to a negative exponent which will result in a negative number when taking the logarithm of. Positive numbers are easy to discuss in everyday life so the negative sign is used to correct the resulting negative log number to be positive.

Figure 1: Elasped Time vs. (T-RT)