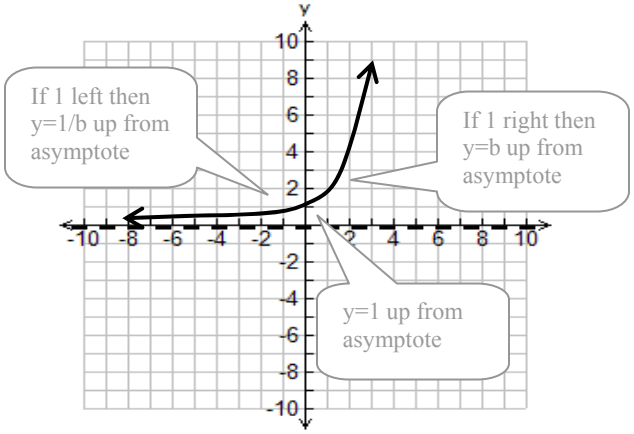
4.12 – Review of Exponential and Logarithmic Functions

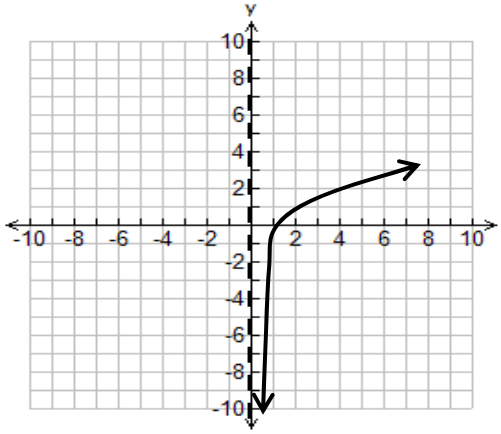
Exponent rules and conventions;

Multiplying exponents with identical bases	$a^n \cdot a^m = a^{n+m}$
Dividing exponents with identical bases	$a^n \div a^m = a^{n-m}$
Power of a power	$(a^n)^m = a^{n \cdot m}$
Power rule	$(ab)^n = a^n \cdot b^n$
Zero exponent	$a^0 = 1$
Negative exponents	$a^{-n} = \left(\frac{1}{a}\right)^n$ or $\frac{1}{a^n}$
Rational exponents	$x^{\frac{a}{b}} = \sqrt[b]{x^a}$ or $\left(\sqrt[b]{x}\right)^a$

Exponential function: $y = a\,b^x + c$



Logarithmic function: $y = log\ _b a + c$



	Exponential	Logarithmic
Base restrictions:	b > 0, x ≠ 1	b > 0, x ≠ 1
Asymptote:	y = 0 (along x-axis)	x = 0 (along y-axis)
Intercepts:	y = 1 (if +a)	x = 1 (if +k)
Domain:	x ε R	x > 0
Range:	y > 0	y ε R
Increases	when b > 1	when b > 1
Decreases	when 0 < b < 1	when 0 < b < 1

Growth/decay formula: $A_f = A_o(base)^{\frac{t}{t_i}}$ where A_f = end A_o = original

Base = ½ if decay or 2 if doubling or ?
t = elapsed time t_i = time interval

Logarithms properties

Basic:
a) $\log_b 1 = 0$
b) $\log_b b = 1$
c) $\log_b b^x = x$
d) $b^{\log_b x} = x$

Operational:
a) $\log_a x^r = r \log_a x$
b) $\log_a xw = \log_a x + \log_a w$
c) $\log_a\left(\frac{x}{w}\right) = \log_a x - \log_a w$

Transforming the exponential functions; $g(x) = \boldsymbol{a}\, b^{(x-\boldsymbol{h})} + \boldsymbol{v}$ where b > 0

Stretch and reflection by a — Translate horizontally

Transforming the logarithmic functions; $g(x) = \boldsymbol{a}\, log_B(x - \boldsymbol{h}) + \boldsymbol{v}$ where b>0 & b≠1

Stretch and reflection by a — Translate horizontally — Translate vertically

Formulii that make use of logarithms: $M = \log\left(\frac{I}{I_o}\right)$ $L = 10\log\left(\frac{I}{I_o}\right)$ $pH = -\log\left[H^+\right]$

4.12 – Review of Exponential and Logarithmic Functions Practice Questions

1. Complete the following table

Exponential form	$3^4 = 81$		$5^3 = 125$		$y = 6^2$	
Logarithmic form	$4 = \log_3 81$	$\frac{1}{2} = \log_{25} 5$		$-3 = \log_{\frac{1}{2}} 8$		$y = \log 10$

2. Simplify and evalutate as exact answer when possible.

a) $\log_5 15 - \log_5 3$ b) $\log_2 48 - \log_2 6$ c) $\log_6 9 + \log_6 4$
d) $\log_6 4 + \log_6 3 + \log_6 3$ e) $\log_3 45 - \log_3 5 + \log_3 3$ f) $\log_8 48 + \log_8 4 - \log_8 3$
g) $\log_3 12 - \log_3 4$ h) $\log_3 9^3$ i) $\log_3 3 + \log_5 1$
j) $\log_5 125^4$ k) $3\log_2 16 - \log_2 8$ l) $\log_3 81 - 2\log_5 125$
m) $\frac{\log_{10} 27}{\log_{10} 3}$ n) $3\log_4 16 + \log_5 (1/5)$ o) $\frac{\log_2 16}{\log_2 8}$

3. Solve to 2 decimal places.

a) $\log_4 20$ b) $\log_6 0.25$ c) $\log_2 10$ d) $5^{3x+4} = 25$
e) $\log_7 15$ f) $10^{2x} = 496$ g) $\log_5 0.5$ h) $\log_9 \sqrt{6}$
i)) $6^x = 55$ j) $\log_6 0.20$ k) $\log_5 250$ l) $2^x = 0.125$
m) $3\log 100$ n) $2\log_3 10$ o) $3^{5-2x} = 875$ p) $12^x = (4)(8^{2x})$
q) $\log x = 2 \log 3 + 2 \log 2$ r) $2^x - 1 = 4$ s) $3\,(2^x) = 18^{x-1}$ t) $8^{3x+1} = 64^{2x}$
u) $\log_2 (x+2) = 3 - \log_2 x$ v) $\log_2 x + \log_2 (x+2) = 3$ w) $\log_2 (2x+2) - \log_2 (x-1) = 3$

4. Sketch the following, clearly indicate asymptote, and the key points.

a) $f(x) = 3^x - 1$ b) $g(x) = (\frac{1}{4})^x + 2$ c) $h(x) = \log_{10}(-3x)$
d) $m(x) = (3)2^x + 3$ e) $y = \log_5 x$ f) $h(x) = -3\,(\frac{1}{4})^x - 1$
g) $y = 4^{x-3} + 2$ h) $y = -(2)^x + 1$ i) $y = \log_{1/2} x$
j) $y = -2\log_3(x-1) + 2$ k) $f(x) = \log_1 x$ l) $h(x) = 3^{2x-4} + 1$
m) $y = \log_2 x$ n) $y = 3\log_4(x+2) + 1$ o) $h(x) = \frac{1}{2} \log_{10}(x)$
p) $f(x) = \log_{0.1} x$ q) $y = (-2)3^x$ r) $y = 2^{-x} + 1$
s) $y = -\frac{1}{2}\log_3(x+1) - 2$ t) $y = -\log_2(\frac{1}{2}(x+1)) - 3$ u) $y = \log_3(-2x+4) + 1$

5. A bamboo plant increases its height by 12% every day. How tall will the plant be after 4 weeks given it starts at a height of 40 cm?

6. It is said that a car depreciates 10% per year. Using this rate, how long will it be before a new car is worth half of its original value?

7. In 1980 a towns' population was 100000. In 2007 it was 220000. Estimate what the population was will be 2020.

8. If paid an interest rate of 5.75% /a compounded annually, come up with an equation to model the total amount as a function of n years. Use an exponential model to determine total amount in your account after 25 years on a $8000 investment.

9. The half life of carbon-14 (C_{14}) is about 5760. If a bone sample only has 15% C_{14} remaining, determine the age of the bone.

10. Compare the magnitude of 2 earthquakes. One measures 4 while the other measures 6.5 on the Richter scale.

11. Find the pH of a liquid with an $[H^+]$ of 8.7×10^{-2} mol/L

12. A babies scream is 1 000 000 000 times more intense than a sound you can just hear. What is the measure of this scream loudness in decibel?

13. The Ph of water in a small lake has dropped from 6.8 to 4.8 over the last three years. How many more times as acidic is the lake now compared to 3 years ago?

14. Find the $[H^+]$ of water which has a pH of 6.80.

15. A culture initially has 2000 bacteria. If the population doubles every 4 hours, determine when the population will be 512000.

16. The half-life or radium is 1620 years. If a laboratory initially has 5g of radium, determine how much they would have in 200 years. How many years would it take until the laboratory only had 4g remaining?

17. When bacteria grow by binary fission, the generation time is the time interval required for the cells (or bacteria population) to divide. The formula for generation time is $G = \dfrac{t}{3.3 \log \dfrac{b}{B}}$,

    where G is the generation time, t is the time interval in hours or minutes, b is the number of bacteria at the end of a time interval, and B is the number of bacteria at the beginning of a time interval.

    a) What is the generation time of a bacterial population that increases from 10^3 cells to 10^6 cells in five hours of growth?
    b) What would be the increase in population if the growth interval was 4 hours and the generation time was 18 minutes?

18. The following tables document European and North American population over the past 250a.

Europe	
Year	Population (in millions)
1750	163
1800	203
1850	276
1900	408
1950	547
2000	729

North America	
Year	Population (in millions)
1750	2
1800	7
1850	26
1900	82
1950	172
2000	305

a) Find the average rate of change in European population from 1750 to 1800.
b) Find the average rate of change in European population from 1950 to 2000.
c) Compare and comment on your answers from a & b.
d) Find the average rate of change in North American population from 1800 to 1850.
e) Find the average rate of change in North American population from 1950 to 2000.
f) Compare and comment on your answers from e & f.
g) From 1800 to 1850 the average rate of change for populations was greater in Europe than in North America, however the change in North American population was far more dramatic. Explain.
h) Find equations to model both populations.
i) Use equations to predict both populations in 2050.
j) If the growth trends continue and the models hold true, when will the populations be the same?

Answers 2. a) 1 **b)** 3 **c)** 2 **d)** 2 **e)** 3 **f)** 2 **g)** 1 **h)** 6 **i)** 1 **j)** 12 **k)** 9 **l)** -2 **m)** 3 **n)** 5 **o)** 4/3 **3. a)** 2.16 **b)** -0.77 **c)** -0.52 **d)** -2/3 **e)** 1.39 **f)** 1.35 **g)** -0.43 **h)** 0.41 **i)** 2.24 **j)** -0.90 **k)** 3.43 **l)** -3 **m) 6 n)** 4.19 **o)** -0.58 **p)** 0.83 **q)** 72 **r)** 2.32 **s)** 1.82 **t)** 1 **u)** 2 **v)** 2 **w)** 2 **5.** 955cm **6.** 6.6a or 6a 7months **7.** $P(t)=100000(1.026)^{t-1980}$, 279000 **8.** $32366.77 **9.** 15765a **10.** 316x more intense **11.** +1 (very acidic) **12.** 90dB **13.** 100x more acidic **14.** $1.58\text{x}10^{-7}$ **15.** 32h **16.** 4.59g, 520a **17. a)** 30.3min **b)** population is 10^7 so increase is $10^7 - 10^4$ **18. a)** 0.8m/a **b)** 3.79m/a **c)** 0.38m/a **d)** 5 fold increase **e)** 7 fold increase **f)** 2.77 m/a **g)** The rate is higher because many more people where added to the population. But from a percent point of view North America growth was much more significant; it is just that they started with smaller numbers. **h)** $E(y)=0.003006(1.006221)^y$, $A(y)=2.27385\text{x}10^{-12}(1.016075)^y$ or $NA(y)=2.9799(2.2193)^{(y-1750)/50}$ **i)** 999B, 358B **j)** around 2155

4.12 - Sketching Practice Sheets