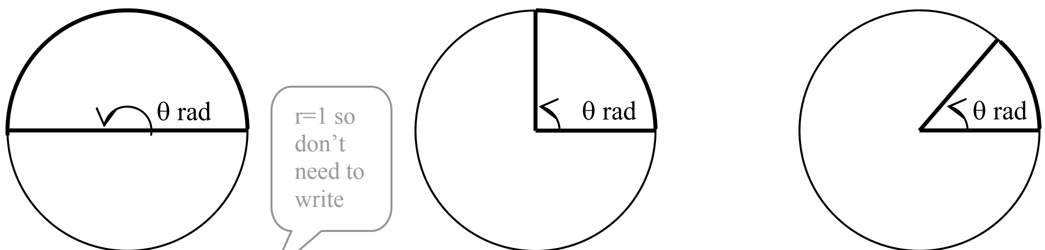
5.1 – Radians

Radians are a unit of measure used in a circle to relate the sector angle to the sector arc length. It is derived from the fact that the circumference of a circle is related to the radius of the circle by an exact ratio symbolized by π.

Ex. Circumference = 2 π radius or C = 2πr

As circumference measures the distance around the entire circle, if we want only half or a quarter of this distance then we would divide the other side of the formula accordingly. We might consider a circle with a radius of 1 to simplify the following examples.

Ex. a) Half circle b) Quarter circle c) Eight Circle



So an arc that is half the circumference relates to π and hence this represents 180° angle

$\frac{C}{2} = \frac{2\pi r}{2}$ *or* $\frac{C}{2} = \pi$

r=1 so don't need to write

$\frac{C}{4} = \frac{2\pi}{4}$ *or* $\frac{C}{4} = \frac{\pi}{2}$

π/2 = 90°

$\frac{C}{8} = \frac{\pi}{4}$

π/4 = 45°

Arc length is the term given to this partial circumference measured between two radii.

A sector angle is measured between these two radii that make the arc as shown above.

Now we can make this substitution back into the circumference formula and rearrange it to give us a useful formula to relate the arc length, radius and sector angle. Leaving the angle in the second form creates this new unit (i.e. radians).

Ex. $C = 2\pi r$ *or* $a = 2\pi r$

so $\frac{a}{r} = 2\pi$

radian angle

This formula tells us that dividing arc length by radius of a circle will give us the sector angle in radians

or $\frac{a}{r} = \theta$ *radians*

$$\boxed{\theta = \frac{a}{r}}$$

360° was chosen to relate to 365 days a year. So 1° corresponds to one day. Luckily 360 is a highly divisible number making many other calculations simpler. Is it strange that 24 hours a day goes in exactly 15 times? I guess everything would have been easy if the Earth took 100 days to go around the sun and there were 10 hours in a day.

Ex. Rad is short form unit used for radians.

Using the radian unit provides a quick and exact way to relate sector angles and length. But as calculators became more prevalent degrees soon became a more popular measure, one can use the fact that there are 360° in a circle which corresponds to one complete rotation (2π radians) to come up with the following way to convert between the two types of units.

Ex. 360° = 2π rad or

$$\boxed{180° = \pi \text{ rad}}$$

Your calculator can operate in either mode DEG or RAD. Many mistakes are made when entering operating in the wrong mode as calculator thinks the number is in one measure while you may think its in another.

Use this ratio to set up your proportion for converting

Example 1: Calculate the sector angle given the arc length and radius

a) a = 12 cm & r = 6 cm

$$\theta = \frac{a}{r} \quad so \quad \theta = \frac{12}{6}$$

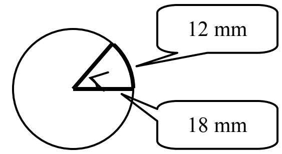
$$\theta = 2\ rad$$

b) a = 15 m & r = 8 m

$$\theta = \frac{14}{8}$$

$$\theta = 1.75\ rad$$

c)



$$\theta = \frac{12}{18}$$

$$\theta = 0.67\ rad$$

Example 2: Calculate the arc length, given the radius and sector angle.

a) r = 10 cm & θ = 2.5 rad

$$\theta = \frac{a}{r} \quad so \quad a = \theta r$$

$$a = 2.5(10)$$

$$a = 25cm$$

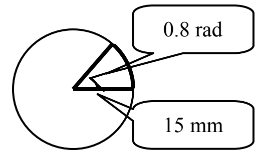
b) r = 4 m & θ = 5 rad

$$a = \theta r$$

$$a = 4(5)$$

$$a = 20m$$

c)



$$a = \theta r$$

$$a = 0.8(15)$$

$$a = 12mm$$

Example 3: Convert from radians to degrees.

a) $3\ rad$

$$\frac{180^\circ}{\pi\ rad} = \frac{x}{3\ rad}$$

$$x = \frac{480^\circ\ rad}{\pi\ rad}$$

$$x = 153^\circ$$

Set up the proportion using the ratio. Notice that the first example includes units, which will cancel out to leave only degrees

b) $4.5\ rad$

$$\frac{180}{\pi} = \frac{x}{4.5}$$

$$x = \frac{810}{\pi}$$

$$x = 258^\circ$$

c) $\frac{2\pi}{3}\ rad$

$$\frac{180}{\pi} = \frac{x}{\frac{2\pi}{3}}$$

$$\frac{180}{\pi} = \frac{3\ x}{2\pi}$$

$$x = \frac{(180)(2\pi)}{3\pi}$$

$$x = 120^\circ$$

Useful to memorize the common conversions

$180° = \pi$ rad

$90° = \frac{\pi}{2}$ rad

$60° = \frac{\pi}{3}$ rad

$45° = \frac{\pi}{4}$ rad

$30° = \frac{\pi}{6}$ rad

Example 4: Convert from degrees to radians

a) 45º

$$\frac{180^\circ}{\pi\ rad} = \frac{45^\circ}{x\ rad}$$

$$x = \frac{45^\circ\ \pi\ rad}{180^\circ}$$

$$x = \frac{\pi}{4}\ rad$$

$$x = 0.785\ rad$$

Degree unit cancel out to leave only radians

b) 210º

$$\frac{180}{\pi} = \frac{210}{x}$$

$$x = \frac{210\pi}{180}$$

$$x = \frac{7\pi}{6}\ rad$$

As exact answer

$$x = 3.67\ rad$$

As approximate round decimal answer

Example 5: A swing has a radius length of 3m. If a child swings through an angle of 135° how far did he/she travel to the nearest tenth of a meter?

Convert to radians: $\frac{3\pi}{4}$ Distance is arc length so

$$\frac{3\pi}{4} = \frac{a}{3}$$

$$\frac{9\pi}{4} = a$$

$$a = 7.1m$$

∴The swing rotates through about 7.1m

Word question. Word answer.

5.1 – Radians Practice Questions

1. Convert to exact radians.

a) 20°	b) 100°	c) -90°	d) 15°
e) 50°	f) 200°	g) 72°	h) -30°
i) 60°	j) 2000°	k) -45°	l) 450°
m) 135°	n) 540°	o) -60°	p) 360°
q) 180°	r) 18°	s) 1200°	t) 25°

2. Convert to degrees.

a) 2π rad	b) π/3 rad	c) 2π/3 rad	d) π/4 rad
e) -3π/4 rad	f) -π rad	g) 7π/6 rad	h) 7π/2 rad
i) π/2 rad	j) 4π/3 rad	k) -3π/2 rad	l) 11π/4 rad
m) 5π/4 rad	n) -7π/8 rad	o) 5π/12 rad	p) -π/3 rad
q) 0.35 rad	r) -0.52 rad	s) -2.5 rad	t) 4.56 rad

3. Calculate the arc length given; a) radius of 12cm and sector angle of 75°
   b) radius of 8m and sector angle of 185°
   c) radius of 18mm and sector angle of π/6
4. Calculate sector angle given; a) radius of 12cm and arc length of 10cm
   b) radius of 8m and arc length of 20m
   c) radius of 5mm and arc length of 40mm
5. If you had to advise another student of quick way to covert from degrees to radians, what would you show them?
6. Discuss the pro and cons of working in degrees. Radians?
7. A 100cm long pendulum swings through an arc of 5cm. Find the angle it swings through in; a) radians b) degrees.
8. If an 80cm door rotates open through 2 radians, what arc distance did the outer edge of the door travel. What area of floor does the door swing over?
9. Given the earth rotates on its axis once every 24 hours, how long does it take to rotate through; a) 120° b) 4π/3 radians
10. Determine the angular velocity, in radians per second, of a 26cm diameter bike tire rotating 3 revolutions per second.
11. The revolving restaurant in the CN Tower completes one rotation every 5/6 of an hour. If Paul ate dinner from 19:15 to 21:35 through what exact radian angle did he rotate through while eating?
12. A 32m diameter Ferris wheel makes two revolutions every minute.
    a) Determine its angular velocity in radians per second.
    b) If a ride lasts 3 minutes, how far does a rider travel?
13. A car tire with a radius of 20cm turns with an angular velocity of 8 rad/s.
    a) Express its angular velocity in revolutions per minute.
    b) Calculate, the distance the tire will roll in 10 seconds.

Answers 1. a) π/9 **b)** 5π/9 **c)** -π/2 **d)** π/12 **e)** 5π/18 **f)** 10π/9 **g)** 4π/10 **h)** -π/6 **i)** π/3 **j)** 100π/9 **k)** -π/4 **l)** 5π/2 **m)** 3π/4 **n)** 3π **o)** -π/3 **p)** 2π **q)** π **r)** π/10 **s)** 20π/3 **t)** 5π/36 **2. a)** 360º **b)** 60º **c)** 120º **d)** 45º **e)** -135º **f)** -180º **g)** 210º **h)** 630º **i)** 90º **j)** 240º **k)** -270º **l)** 495º **m)** 225º **n)** 157.5º **o)** 75º **p)** -60º **q)** 20º **r)** -30º **s)** 143º **t)** 261º
3. a) 15.7cm **b)** 25.8m **c)** 9.4mm **4. a)** 0.83rad or 47º **b)** 2.5rad **c)** 8rad **5.** multiple by 180/π or 57.3
7. a) 0.05rad **b)** 2.9º **8.** 160cm, area = 80^2 or 6400 cm^2 (π's cancel out is as a benefit of working in radians) **9. a)** 8h **b)** 16h **10.** 6π rad **11.** 28π/5 or 1008° **12. a)** π/15 rad/s **b)** 192π or 603m **13. a)** 76.4 rev/min **b)** 1600cm