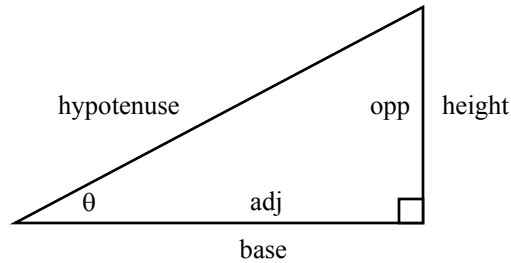


5.2 – Trigonometric Ratios for Any Angle (CAST)

Recall the basic definitions for the common right triangle trigonometric ratios;

Height, base and hypotenuse where the original reference s with opposite and adjacent introduced to develop the memory aid SohCahToa



$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{height}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{base}}{\text{hypotenuse}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{\text{height}}{\text{base}} = \frac{\sin}{\cos}$$

The corresponding reciprocal trigonometric ratios would then be;

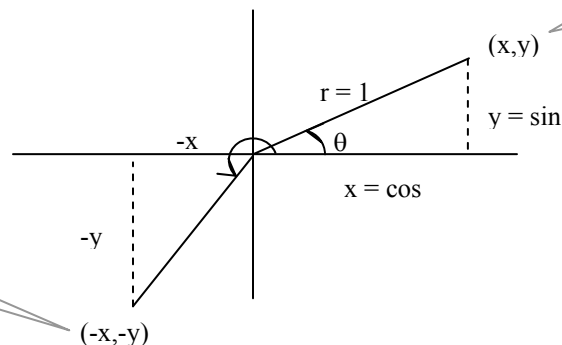
$$\csc \theta = \frac{1}{\sin \theta} = \frac{\text{hypotenuse}}{\text{height}} \quad \sec \theta = \frac{1}{\cos \theta} = \frac{\text{hypotenuse}}{\text{base}} \quad \cot \theta = \frac{1}{\tan \theta} = \frac{\text{base}}{\text{height}} = \frac{\cos}{\sin}$$

Reciprocal ratios would have proven useful to use before tables and calculator

Plotting these concepts on a Cartesian grid and using the hypotenuse as a radial (i.e. radius) arm with unit length, one can rotate this radial arm through various angles to get gain insight in the impact on the basic trigonometric ratios for any angle.

Ex.

Notice that same ratios can occur in this quadrant based on reflections from 1st quadrant. It is just that the signs might change.



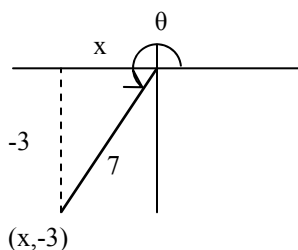
This radius is called a terminal arm because it ends at coordinates (x,y)

S	A
T	C

CAST is an acronym that can help one remember what the sign of the basic trigonometric ratio would be in any of the four quadrants. The letter corresponds to the ratio which is positive in the given quadrant. One starts to label in the 4th quadrant and rotates counterclockwise.

Example 1: If $\sin \theta = -3/7$ and terminal arm is located in quadrant III, find other ratios.

Draw a sketch of scenario based on information given



$$\begin{aligned} 7^2 &= (-3)^2 + x^2 \\ 49 - 9 &= x^2 \\ \sqrt{40} &= x \\ \pm 2\sqrt{10} &= x \\ -2\sqrt{10} &= x \end{aligned}$$

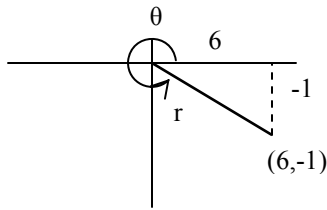
$$\therefore \cos \theta = \frac{-2\sqrt{10}}{7}$$

$$\therefore \tan \theta = \frac{3}{2\sqrt{10}}$$

Use Pythagorean Theorem to calculate length x.. We know from diagram to use -x.

Did not ask for θ° , but if did could use \sin^{-1} of $-3/7$ on calculator to get 25°

Example 2: Given the point P(6,-1) lies at the end of a terminal arm, find all the basic trigonometric ratios and angle arm is rotated through.



$$r^2 = 6^2 + (-1)^2$$

$$r^2 = 37$$

$$r = \sqrt{37}$$

so: $\sin \theta = \frac{-1}{\sqrt{37}}$ $\cos \theta = \frac{6}{\sqrt{37}}$ $\tan \theta = \frac{-1}{6}$

and: $\theta = -9.4^\circ$ $\theta = +9.4^\circ$ $\theta = -9.4^\circ$

$$\theta = 360 - 9.4^\circ$$

$$\theta = 351^\circ$$

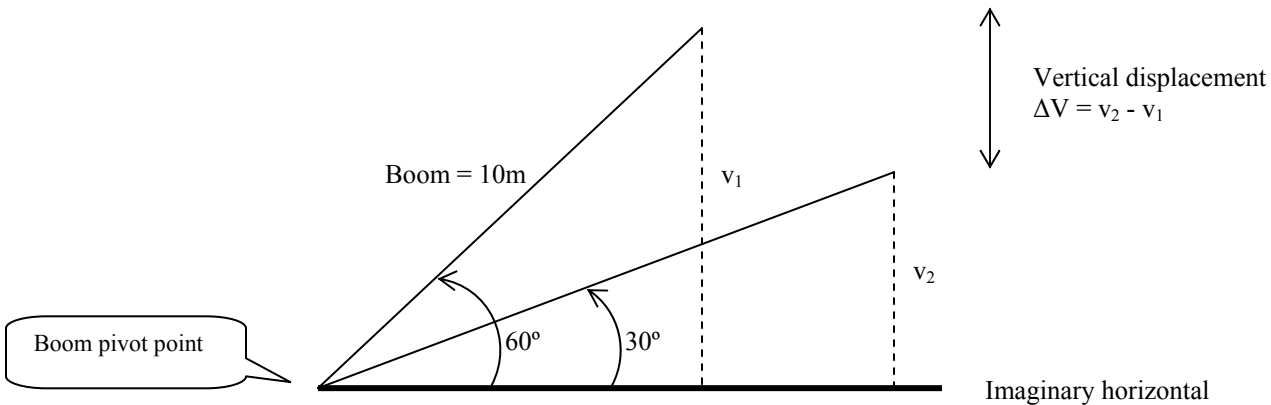
Think height over hypotenuse

Think ratio of height to base

There are two equivalent ratios for each, so one might have to adjust the calculator answer using diagram to match to correct angle

Make sure your calculator is in degree mode

Example 4: A crane with a 10m boom, lower its boom from 60° to 30° . Find the vertical displacement the end of the boom travels through.



Set up ratios: $\sin 60^\circ = \frac{v_1}{10}$ $\sin 30^\circ = \frac{v_2}{10}$

$$v_1 = 10 \sin 60^\circ$$

$$v_2 = 10 \sin 30^\circ$$

Approximately (rounded decimal)

Vertical displacement: $\Delta V = 10 \sin 60^\circ - 10 \sin 30^\circ$

$$= -3.77$$

Negative because moved down

Exact using values from unit circle (lesson 5.3)

Vertical displacement: $\Delta V = 10 \sin 60^\circ - 10 \sin 30^\circ$

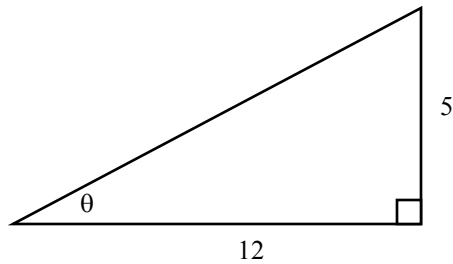
$$= 10 \frac{\sqrt{3}}{2} - 10 \frac{1}{2}$$

$$= 5(\sqrt{3} - 1)$$

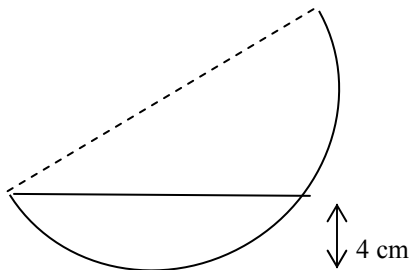
\therefore The boom end is vertically displaced about 3.8m

5.2 – Trigonometric Ratios for Any Angle Practice Questions

1. Given the triangle below express all basic and reciprocal trigonometric ratios.



2. Write all six trigonometric ratio given $\sin x = -3/5$ in the 3rd quadrant.
3. Find value of θ to nearest degree on interval $0^\circ \leq \theta \leq 360^\circ$. Use CAST to help get both angles.
- | | | | |
|--------------------------|-------------------------|--------------------------|-------------------------|
| a) $\sin \theta = 0.53$ | b) $\sec \theta = 1.58$ | c) $\cot \theta = 0.25$ | d) $\tan \theta = 6.81$ |
| e) $\cos \theta = -0.86$ | f) $\csc \theta = 3.27$ | g) $\sin \theta = -0.15$ | h) $\sec \theta = -2.3$ |
| i) $\tan \theta = -1.7$ | j) $\cos \theta = 0.21$ | k) $\cot \theta = 0.71$ | d) $\cos \theta = 0.5$ |
4. Given point (6,-8) is point on a terminal arm in standard calculate angle of rotation.
5. A 45m tall tree cast a 12m shadow. Calculate the angle the Sun makes with the ground at this time.
6. A hemispherical bowl of diameter 20cm contains some liquid with a depth of 4cm. Through what angle, with respect to the horizontal, may the bowl be tipped before the liquid begins to spill out. See diagram below.



Answers 1. a) $\sin \theta = 5/13$, $\cos \theta = 12/13$, $\tan \theta = 5/12$, $\csc \theta = 13/5$, $\sec \theta = 13/12$, $\cot \theta = 12/5$ 2. $\sin \theta = -3/5$, $\cos \theta = -4/5$, $\tan \theta = 4/3$, $\csc \theta = -5/3$, $\sec \theta = -5/4$, $\cot \theta = 3/4$ 3. a) 32° or 309° b) 51° or 309° c) 76° or 256° d) 82° or 262° e) 149° or 211° f) 18° or 162° g) 189° or 351° h) 116° or 244° i) 120° or 300° j) 78° or 282° k) 55° or 235° l) 60° or 300° 4. 1.3 rad or 307° 5. 75° 6. 37°