5.4 – Equivalent Trigonometric Ratios (Co function identities)

Recall the complimentary property of sine and cosine ratios. This is also referred to as the co function identity because it relates one function (i.e. sin θ) to another.

$$\sin\theta^\circ = \cos(90-\theta^\circ) \quad \textit{and} \quad \sin(90-\theta^\circ) = \cos\theta^\circ$$

or

$$\sin\theta^\circ = \cos(\frac{\pi}{2}-\theta^\circ) \quad \textit{and} \quad \sin(\frac{\pi}{2}-\theta^\circ) = \cos\theta^\circ$$

Given in radians so use π/2 radians

Example 1: Write the equivalent co function for the following;

a) $\cos 51^\circ$

Verify this on the calculator

$= \sin(90^\circ - 51^\circ)$

$= \sin(39^\circ)$

b) $\sin 135^\circ$

$= \cos(90^\circ - 135^\circ)$

$= \cos(-45^\circ)$

c) $\sin\frac{\pi}{6}$

$= \cos\left(\frac{\pi}{2}-\frac{\pi}{6}\right)$

$= \cos\left(\frac{3\pi}{6}-\frac{\pi}{6}\right)$

Use fraction rules to get common denominator

$= \cos\left(\frac{2\pi}{6}\right)$

$= \cos\left(\frac{\pi}{3}\right)$

Using the "Unit Circle one can also show the supplementary property;

This is why sin θº is ambiguous between 0º < θ < 180º

$$\sin\theta^\circ = \sin(180-\theta^\circ) \quad \textit{and} \quad \cos\theta^\circ = -\cos(180-\theta^\circ)$$

or

$$\sin\theta^\circ = \sin(\pi-\theta^\circ) \quad \textit{and} \quad \cos\theta^\circ = -\cos(\pi-\theta^\circ)$$

Example 2: Write the equivalent function for the following using supplementary property;

a) $\cos 51^\circ$

Verify this on the calculator

$= -\cos(180^\circ - 51^\circ)$

$= -\cos(129^\circ)$

b) $\sin 135^\circ$

$= \sin(180^\circ - 135^\circ)$

$= \sin(45^\circ)$

c) $\sin\frac{\pi}{6}$

$= \sin(\pi - \frac{\pi}{6})$

$= \sin(\frac{6\pi}{6} - \frac{\pi}{6})$

$= \sin(\frac{5\pi}{6})$

5.4 – Equivalent Trigonometric Ratios Practice Questions

1. Write equivalent co function.

a) $\sin\frac{\pi}{12}$ b) $\sin\frac{2\pi}{5}$ c) $\sin\frac{5\pi}{8}$ d) $\sin\frac{5\pi}{12}$

e) $\cos\frac{5\pi}{18}$ f) $\cos\frac{\pi}{9}$ g) $\cos\frac{7\pi}{36}$ h) $\cos\frac{2\pi}{9}$

i) $\sec\frac{2\pi}{3}$ j) $\csc\frac{\pi}{6}$

2. Write 2 equivalent ratios using the same function.

a) $\sin\pi$ b) $\cos\frac{\pi}{2}$ c) $\sin\frac{\pi}{6}$ d) $\cos\frac{2\pi}{3}$

e) $\sin-2\pi$ f) $\cos-\frac{3\pi}{4}$ g) $\sin\frac{\pi}{2}$ h) $\cos\frac{5\pi}{6}$

i) $\csc\frac{\pi}{2}$ j) $\sec-\frac{\pi}{3}$

Answers 1. a) $\cos\frac{5\pi}{12}$ **b)** $\cos\frac{\pi}{10}$ **c)** $\cos-\frac{\pi}{8}$ **d)** $\cos\frac{\pi}{12}$ **e)** $\sin\frac{2\pi}{9}$ **f)** $\sin\frac{7\pi}{18}$ **g)** $\sin\frac{11\pi}{36}$ **h)** $\sin\frac{5\pi}{18}$ **i)** $\csc-\frac{\pi}{6}$ **j)** $\sec\frac{\pi}{3}$ **2.** answer may vary as many equivalent ratios exist. To be sure you can verify your answer compared to the original on a calculator **a)** $\sin 0$, $\sin-\pi$ **b)** $\cos\frac{5\pi}{2}$, $\cos-\frac{\pi}{2}$ **c)** $\sin\frac{5\pi}{6}$, $\sin\frac{17\pi}{6}$ **d)** $-\cos\frac{\pi}{3}$, $\cos\frac{7\pi}{3}$ **e)** $\sin 0$, $\sin 2\pi$ **f)** $-\cos\frac{\pi}{4}$, $\cos\frac{5\pi}{4}$ **g)** $-\sin\frac{3\pi}{2}$, $\sin\frac{\pi}{2}$ **h)** $\cos\frac{17\pi}{6}$, $-\cos\frac{\pi}{6}$ **i)** $-\csc\frac{3\pi}{2}$, $\csc\frac{\pi}{2}$ **j)** $\sec\frac{\pi}{3}$, $\sec\frac{5\pi}{3}$ or $-\sec\frac{2\pi}{3}$, $-\sec\frac{4\pi}{3}$