5.5B – Double Angle Formulas (Sum Identities)

From our sine compound angle formula $\sin(x+y)=\sin x\cos y+\cos x\sin y$

It follows that

$$\begin{aligned}\sin(x+x)&=\sin x\cos x+\cos x\sin x\\ \sin(2x)&=\sin x\cos x+\sin x\cos x\\ &=2\sin x\cos x\end{aligned}$$

From our cosine compound angle formula $\cos(x+y)=\cos x\cos y-\sin x\sin y$

It follows that

$$\begin{aligned}\cos(x+x)&=\cos x\cos x-\sin x\sin x\\ \cos(2x)&=\cos^2 x-\sin^2 x\end{aligned}$$

And given $\cos^2 x=1-\sin^2 x$ then $\cos(2x)=1-2\sin^2 x$

And given $\sin^2 x=1-\cos^2 x$ then $\cos(2x)=2\cos^2 x-1$

Using substitution we can also generate a tangent double angle formula. In summary one can list the following double angle formulas:

$$\sin(2x)=2\sin x\cos x \qquad \begin{aligned}\cos(2x)&=\cos^2 x-\sin^2 x\\ &=2\cos^2 x-1\\ &=1-2\sin^2 x\end{aligned} \qquad \tan(2x)=\frac{2\tan x}{1-\tan^2 x}$$

Example 1: Given $\sin x=\frac{2}{3}$ where $\frac{\pi}{2}<x<\pi$ find

a) sin 2x

$$\begin{aligned}&=2\sin x\cos x\\ &=2\left(\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right)\\ &=-\frac{4\sqrt{5}}{9}\end{aligned}$$

We need to find a ratio for cos x because it was not given. See below.

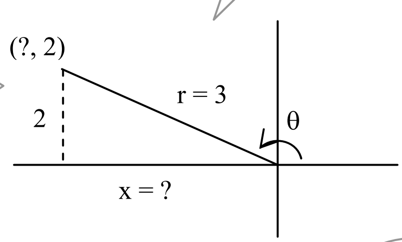
b) cos 2x

$$\begin{aligned}&=1-2\sin^2 x\\ &=1-2\left(\frac{2}{3}\right)^2\\ &=\frac{9}{9}-\frac{8}{9}\\ &=\frac{1}{9}\end{aligned}$$

Makes most sense to use formula with ratio that is given. (i.e. we know sin x)

Draw a sketch of scenario based on information given

Recall that there are always two similar values for each ratio between 0° and 360°. The question state that we should use the one between 90° and 180°, hence draw terminal arm in 2nd quadrant



$$\begin{aligned}3^2&=(2)^2+x^2\\ 9-4&=x^2\\ 5&=x^2\\ \pm\sqrt{5}&=x\end{aligned}$$

$-\sqrt{5}=x$

Diagram shows that x value must be negative one

$\therefore \cos\theta=\frac{-\sqrt{5}}{3}$

Set up cosine ratio that can now substitute

5.5B – Double Angle Formulas Practice Questions

1. Use double angle and sum formulii to each rewrite each of the following as single angles.

a) sin 2x   b) sin 4x   c) sin 3x   d) cos 4x

e) cos 3x   b) cos 5x   c) cos 6x   d) sin 6x

2. If $\sin x = -\frac{3}{5}$ where $\pi < x < \frac{3\pi}{2}$ find   a) sin 2x   b) cos 2x

3. If $\cos\theta = -\frac{5}{13}$ where $\frac{\pi}{2} < \theta < \pi$ find   a) sin 2θ   b) cos 2θ

4. Prove that, for any angle, θ   a) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
b) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

Remember that proofs require left side (LS) right side (RS) format

5. Evaluate the following for x = π/6

a) sin 2x   b) 2 sinx   c) 2 sin x cos x   e) $\cos^2 x - \sin^2 x$ f) cos 2x

Answers 1. a) 2 sin x cos x **b)** $4\sin x \cos^3 x - 4\sin^3 x \cos x$ **c)** $3\cos^2 x \sin x - \sin^3 x$ (see solution below)
d) $\cos^4 x - 6\cos^2 x \sin^2 x + \sin^4 x$ **e)** $\cos^3 x - 3\sin^2 x \cos x$ **f)** $\cos^5 x - 10\sin^2 x \cos^3 x + \sin^4 x \cos x$
g) $\cos^6 x - 15\cos^4 x \sin^2 x + 15\cos^2 x \sin^4 x - \sin^6 x$ **h) 6**$\cos^5 x \sin x - 20\cos^3 x \sin^3 x + 6\cos x \sin^5 x$
2. a) 24/25 **b)** 7/25 **3. a)** -120/169 **d)** -119/169 **3)** set up LS and RS and use the formulas to make substitutions along with general algebraic simplification until you get one side to equal the other.
5.a) √3/2 **b)** 1 **c)** √3/2 **d)** ½ **e)** ½

1c) sin (2x + x)
= sin 2x cos x + cos 2x sin x
= 2 sin x cos x cos x + $(\cos^2 x - \sin^2 x)$ (sin x)
= $2\sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x$
= $3\sin x \cos^2 x - \sin^3 x$