

## 5.8A – Transforming the Trigonometric Functions

Recall the basic transformations format:

$$g(x) = a f[k(x-h)] + v$$

Stretch and reflection by a

Stretch horizontally by 1/k

Translate vertically

Translate horizontally

The same transformations apply to the trigonometric functions. Only that instead of using translation and stretch the periodic terms, phase shift, displacement and amplitude are used.

Ex.  $g(x) = a \sin [k(x-h)] + v$

Amplitude = a

Period =  $2\pi/k$

Horizontal phase shift of h

Vertical displacement of v

**Example 1:** Describe the transformation on the following functions;

a)  $f(x) = 2 \sin (2(x + 45^\circ)) + 5$

Displacement: + 5 (vertically up 5 units)  
 Phase shift:  $-45^\circ$  (horizontally left  $45^\circ$ )  
 Period:  $360^\circ/2$   
 $180^\circ$  (horizontally compressed)  
 Amplitude: 2 (vertically stretched)

b)  $g(x) = -3 \csc \left[ \frac{1}{2} \left( \theta - \frac{\pi}{6} \right) \right] - 12$

Displacement: -12  
 Phase shift:  $+\pi/6$   
 Period:  $2\pi \div 0.5$   
 $4\pi$   
 Amplitude: 3

Given in radian means you will graph in radians

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**Example 2:** Sketch 1 complete cycle for each of the following

a)  $y = 2 \sin(x) + 3$

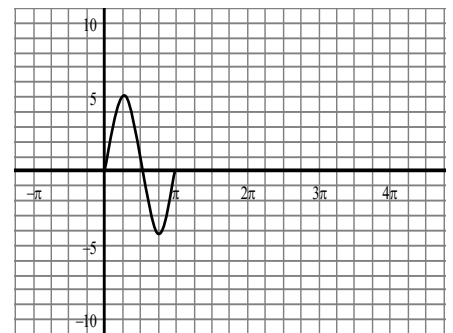
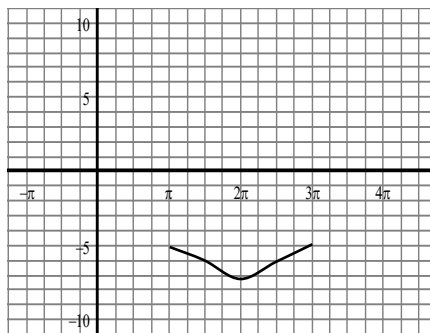
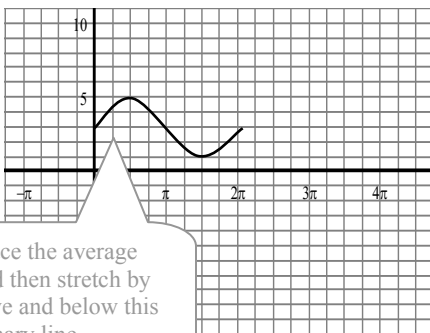
D: + 3  
 PS: n/a  
 P: 360 or  $2\pi$   
 A: 2

b)  $g(x) = \cos(\theta - \pi) - 5$

D: - 5  
 PS: +  $\pi$   
 P:  $2\pi$   
 A: 1

c)  $y = 4 \sin(2x)$

D: n/a  
 PS: n/a  
 P:  $2\pi/2$   
 $\pi$   
 A: 4



### 5.8A – Transforming the Trigonometric Functions Practice Questions

- Describe the transformations (i.e. displacement, phase shift, period, amplitude) on  $g(x) = \cos x$  to get  $h(x) = 2 \cos (2x) - 1$ .
- Sketch 1 complete cycle for each of the following. Use radians unless degrees specified.
 

a) $y = \sin 2x$	b) $y = 3 \cos x - 2$	c) $f(x) = 2 \tan x$
d) $g(x) = 3 \csc(x)$	e) $h(x) = \sec 2(x + \pi)$	f) $y = -2 \sin (x - \pi/2) - 1$
g) $y = 3 \cos (\frac{1}{2} x + \pi/2) - 1$	h) $g(x) = \tan \frac{1}{2} x + 1$	i) $g(x) = 3 \sin (2\theta - 60^\circ)$
- Accurately sketch two complete cycles for  $y = 4 \cos 2x$ 
  - Describe intervals where the function is increasing
  - Describe intervals where the function is decreasing
  - Describe intervals where the function is stationary (neither increasing or decreasing)
  - Calculate average rate of change in the function from
 

i) $-\pi$ to $\pi$	ii) $\pi$ to $2\pi$
iii) $\pi/2$ to $\pi$	iv) $\pi$ to $3\pi/2$
  - Calculate instantaneous rate of change in function at
 

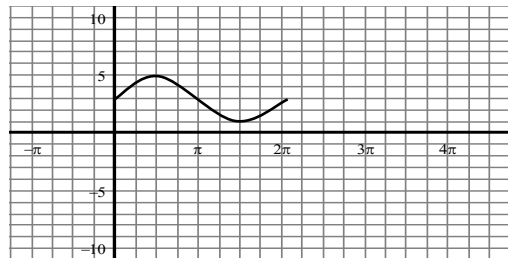
i) $\pi$	ii) $\pi/2$
iii) $\pi/4$	iv) $3\pi/4$
- Accurately sketch  $y = 3 \sin \frac{1}{2} x + 1$  on the interval  $-\pi \leq x \leq 3\pi$ 
  - Describe intervals where the function is increasing
  - Describe intervals where the function is decreasing
  - Determine the maximum and minimum values of the function.
  - Calculate average rate of change in the function from
 

i) $-\pi$ to $\pi$	ii) $\pi$ to $2\pi$
iii) $\pi/2$ to $\pi$	iv) $-\pi$ to $3\pi$
  - Calculate instantaneous rate of change in function at
 

i) $\pi$	ii) $\pi/2$
iii) $-\pi/3$	iv) $2\pi$
- Compare the following graphs by sketching on same axis. Discuss your findings.
 

a) $y = \sin x$	&	$y = \cos (x - \pi/2)$	&	$y = \sin -x$
b) $y = \sin x$	&	$y = -\sin x$	&	$y = \sin -x$
c) $y = \sin 2x$	&	$2 \sin x \cos x$	&	
d) $y = \cos(x)$	&	$y = -\cos x$	&	$y = \cos (-x)$

- Determine the equation of the following function.



**Answers 1.** D=1, PS=0, P= $\pi$ , A=1 **3. a)**  $-\pi/2 < x < \pi$  &  $3\pi/2 < x < 2\pi$  **b)**  $0 < x < \pi/2$  &  $\pi < x < 3\pi/2$  **c)** 0,  $\pi$ ,  $2\pi$   
**d)** i. 0 ii. 0 iii.  $16/\pi$  iv.  $-16/\pi$  **e)** i. 0 ii. 0 iii.  $-32/\pi$  iv.  $32/\pi$  **4. a)**  $-\pi < x < \pi$  **b)**  $\pi < x < 3\pi$  **c)** max  $y=4$ , min  $y=-2$   
**d)** i.  $3/\pi$  ii.  $-3/\pi$  iii.  $3/\pi$  iv. 0 **e)** i. 0 ii.  $4/\pi$  iii.  $4/\pi$  iv.  $-6/\pi$  **5. a)** same graph because sin and cosine are complimentary **b)** 1<sup>st</sup> is vertically flipped, 2<sup>nd</sup> is same **c)** same (double angle formula) **d)** 1<sup>st</sup> is vertically flipped, 2<sup>nd</sup> is same **6.**  $y=2\sin x$

## 5.8A – Transforming the Trigonometric Functions Practice Sheets

