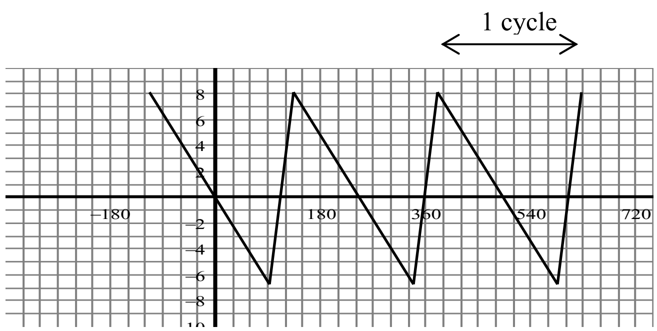
5.9 – Modeling Periodic Behavior

Recall the terminology associated with periodic behavior

Cycle	describes one complete pattern (end up at the same y-value that started with)
Period	describes the horizontal length (x-length) of one cycle
Amplitude	is half the distance between the maximum (peak) and minimum (trough) values of the function. ***Amplitude = (max-min)/2***
Average	is the middle between max and min of the function. ***Average = Min + Amplitude***



You will need to divide 360° or 2π by period to find "**k**" factor

Period = 610 - 390
= 420

So function varies 7.5 above and below its average value

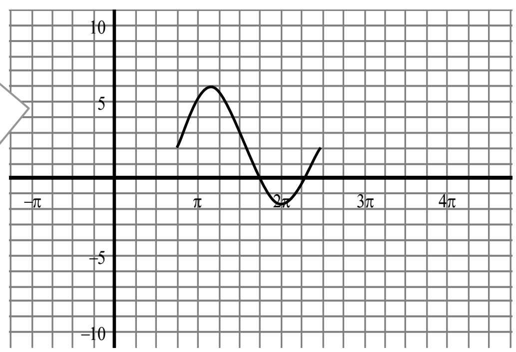
Amplitude = (8 - -7)/2
= 7.5

Average = -7 + 7.5
= 0.5

This will give the displacement

Example 1: Determine the equation of the following using the sine function;

For real life examples it is better to work in degrees because most calculators are set to this default and most people will recognize decimal numbers easier.

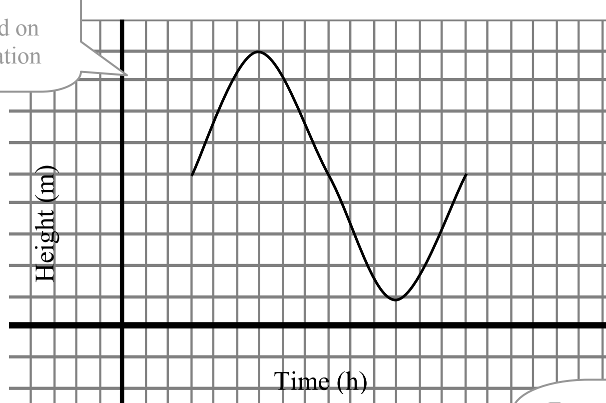


Function to use: sine
Displacement (v): + 2
Phase shift (h): $+3\pi/4$
Period (need k): $k = 2\pi \div 7\pi/4$
$k = 8/7$
Amplitude (a): 4

$$\therefore \quad f(\theta) = 4\sin\left[\frac{8}{7}\left(\theta - \frac{3\pi}{4}\right)\right] + 2$$

Example 2: A 12 hour tide cycle has low tide is 2m and high tide is 14m. Given the tide is at 8m and rising at 3am find function to model tide height t hours after midnight

Sketch the scenario based on given information



Seems to suit scenario best

Function to use: sine
Displacement (v): + 8
Phase shift (h): + 3
Period (need k): $k = 360 \div 12$
$k = 30$
Amplitude (a): 6

$\therefore$ *f(t) = 6 sin 30 (t - 3) + 8*

For real life examples it is better to work in degrees because most calculators are set to this default and most people will recognize decimal numbers easier.

Test out your function to see if it works?

(i.e f(6) should = 14)

Calculator in DEG

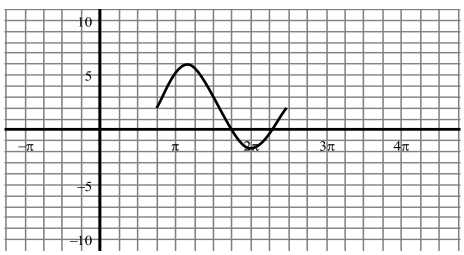
5.9 – Modeling Periodic Behavior Practice Questions

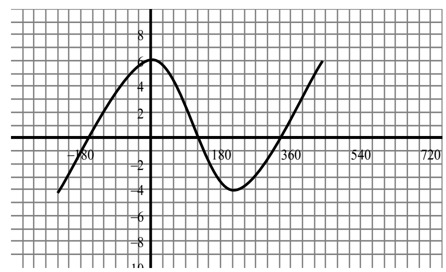
1. In a predator-prey system, the number of predator and the number of prey tend to vary in a periodic manner. In a certain region with cats as predators and mice as prey, the mice population M varied according to the equation $M(t)=110250\sin(\frac{1}{2}\pi t)$, where t is the time in years since January 1996. Graph the function on the interval $0 < t < 2$ and find;

   a) the number of mice when $t = 0.75$, $t = 0.85$.
   b) the average rate of change as t goes from 0.75 to 0.85.
   c) the instantaneous rate of change at $t = 0.85$. Calculator CALC function can help.

2. Determine the equation of the following using most appropriate function.

   a) Sin function with displacement of 4 and period of 180°.
   b) An upside down cosine function with period of 3π, phase shift of π, and amplitude of 7.
   c) Refer to graph below
   d) Refer to graph below





3. A sinusoidal function has an amplitude of 2 units, a period of π, and a maximum at (0,3). Represent the function with an equation in two different ways.

4. A Ferris wheel with a diameter of 50ft rotates every 30 seconds. The vertical position of a person on the Ferris wheel, above and below an imaginary horizontal plane through the center of the wheel can be modeled by the equation $h(t) = 25 \sin 12 t$. Graph the function on the interval from 15 to 30 seconds.

   a) Use equation to determine a person's height, with respect to imaginary plane, at t = 20s.
   b) Use your sketch determine the average rate of change, in the riders height with respect to the middle of the wheel from 24 to 25 seconds.
   c) Find the instantaneous rate of change at 24 seconds
   d) Enter the equation into your graphing calculator and use the CALC features to verify your answers.

5. The depth of water at the end of a pier varies with the tides throughout the day and can be modeled by the equation $D(t) = 1.5\cos(0.575(t-3.5)) + 3.8$, where t is in hours since midnight. Graph the function on the interval $0< t < 10$ and find;

   a) the average rate of change in tide height as t goes form 2 to 5 hours.
   b) the instantaneous rate of change at $t = 5$

Answers **1. a)** 2266, 2568 **b)** 3026 mice/year **c)** 3021 mice/year **2. a)** $y = \sin(2x) + 4$ **b)** $y = -7\cos[3/2(x - \pi)]$ **c)** $y = 4\sin[8/7(x + 3\pi/4) + 2$ **d)** $y = -5\cos[4/5(x + 240)] + 1$ **3.** $y = 2\sin[2(x - \pi/4)] + 1$ and $y = 2\sin[2(x + 5\pi/4)] + 1$ **4. a)** -21.7ft (i.e. below plane) **b)** h(24)=-23.8, h(25)=21.7 so Δh=2.1ft/s **c)** 1.62ft/s **5. a)** d(2)=4.86, d(5)=4.86 so Δd=0m/h **b)** but instant change at t=5 is -0.56m/h

5.9 – Modeling Periodic Behavior Practice Sheets