5.11A – Trigonometric Equations

A ***trigonometric equation*** can be thought of as any equation that contains one or more trigonometric functions. The equation may but usually is not an identity.

Ex. $x + 1 = 0$ is a linear equation
$\sin x + 1 = 0$ is a trigonometric equation

When solving trigonometric equations one treats the unknown variables like they would in any other equation. The difference comes in the last step when using the inverse function to find the radian or degree measure. As trigonometric functions are periodic there can be several solutions that fall within specified interval.

Step of solving method:
a) re-arrange to isolate unknown
b) factor if necessary
c) use zero principle in necessary
d) use inverse trigonometric function to get angle (deg or rad)
e) adjust answer to suit interval using CAST or sketches
f) check answer by substituting back into original equation

Example 1: Solve the following for the interval $0° \le \theta \le 360°$

Interval specified in degrees means your final answer should be in degrees

a) $2\sin\theta = 1$

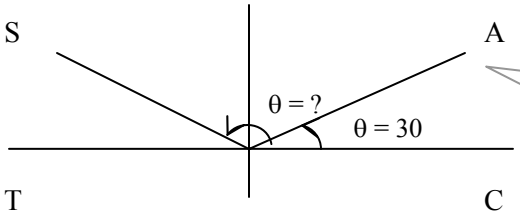
$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

Use either your calculator $\sin^{-1}$ or the unit circle to find θ

Calculator gives θ = 30°

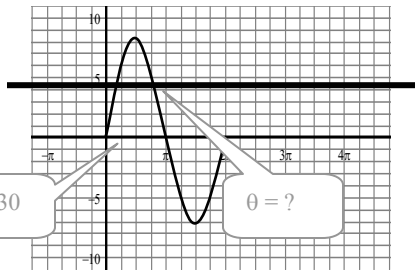
But there could be other angles in the interval specified?



One can use a crude sketch of the unit circle and the CAST acronym to help you figure out where else between 0 and 360 that sin is +½ .

Reflecting the 30 arm one see that 180-30 gives 150.

sin θ = +½



OR one could use a crud sketch of the sin function to estimate where else between 0 and 360 that sin is +½

Therefore θ = 30° or 150°

Both answers verify on calculator
(i.e. sin 30° = ½) and
(i.e. sin 150° = ½)

b) $\sin\theta\cos\theta + \sin\theta = 0$

$\sin\theta\cos\theta + \sin\theta = 0$

$\sin\theta(\cos\theta + 1) = 0$

$\cos\theta + 1 = 0$ or $\sin\theta = 0$

$\cos\theta = -1$

θ = 180° or θ = 0°,180°, 360°

Therefore θ = 0°, 180°, or 360°

All three answer work in specified interval.

This time answer in radians. Mind the interval

Example 2: Solve the following for the interval $\pi \le x \le 2\pi$

a) $2\cos^2 x - \cos x - 1 = 0$

When factoring just think of as $2x - x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$ or $\cos x - 1 = 0$

$2\cos x = -1$ $\cos x = 1$

$\cos x = -\frac{1}{2}$

Calculator gives 60, but this is not within the specified interval, so need to adjust.

$x = 240°$ $x = 360°$

$\therefore x = 4\pi/3$ or 2π

Convert final answer to radians

2π is still within interval because was less than or equal to sign

b) $6\cos^2 x - \sin x = 5$

$6\cos^2 x - \sin x - 5 = 0$

$6(1 - \sin^2 x) - \sin x - 5 = 0$

$6 - 6\sin^2 x - \sin x - 5 = 0$

$-6\sin^2 x - \sin x + 1 = 0$

$-(6\sin^2 x + \sin x - 1) = 0$

$6\sin^2 x + 3\sin x - 2\sin x - 1 = 0$

$3\sin x(2\sin x + 1) - 1(2\sin x + 1) = 0$

$(2\sin x + 1)(3\sin x - 1) = 0$

$2\sin x + 1 = 0$ or $3\sin x - 1 = 0$

$\sin x = -\frac{1}{2}$ $\sin x = 1/3$

$x = -30°$ $x = 20°$

$x = 210°, 330°$ $x = 160°$

$\therefore x = 7\pi/6$ or $11\pi/6$

Conversion involves some rounding unless you want to try formulas to get exact.

5.11A – Trigonometric Equations Practice Questions

1. What is the difference between reciprocal and inverse trigonometric functions? Use a sinusoidal function along with correct math syntax to illustrate and explain.

2. Find all solutions on interval $0° \le x \le 360°$

   a) $\sin x = \frac{\sqrt{3}}{2}$ b) $2\cos x = -\sqrt{3}$ c) $\tan x = -0.237$

   d) $4\sin^2 x - 1 = 2$ e) $\sin^2 x = 6\sin x - 9$ f) $2\sin x \tan x - \tan x - 2\sin x + 1 = 0$

3. Find all solutions on interval $0 \le x \le 2\pi$

   a) $3\sin x = 2\cos^2 x$ b) $4\cos^2 x - 3 = 0$ c) $\cos x = 2\sin x \cos x$

   d) $2\sin^2 x + 5\sin x + 3 = 0$ e) $4\sin^2 x - 1 = 0$ f) $(\cos x)(\sin x) = \cos x$

4. Solve ***sin x = 1*** algebraically in degrees. Verify your answer by graphing the corresponding function $f(x) = \sin x - 1$ and using the trace function on your calculator to find the zeros.
5. Solve ***$2\sin^2 x + \sin x - 1 = 0$*** algebraically in radians. Verify your answer(s) by finding the zeros on the TI-83+ calculator by graphing the corresponding function.
6. Explain how one solve ***cos 2x >1/2*** .

Answers 1. reciprocal is $(\sin x)^{-1} = 1/\sin x = \csc x$ where as inverse is $\sin^{-1} x$ and is used when you want to undo sine function. **2. a)** 60°, 120° **b)** 150°, 210° **c)** 167°, 347° **d)** 60°, 120° **e)** no solution **f)** 30°, 150° & 45°, 225° **3.a)** $\pi/6, 5\pi/6$ **b)** $\pi/6, 11\pi/6$ **c)** ? **d)** $3\pi/2$ **e)** $\pi/6, 5\pi/6, 7\pi/6, 11\pi/6$ **f)** $\pi/2, 3\pi/2$ **3.** x=90 or x= 90°+360°n **4.** $x = \pi/6 + 2\pi n$ or $x = 5\pi/6 + 2\pi n$ or $x = 3\pi/2 + 2\pi n$ **5.** One could solve $\cos 2x - \frac{1}{2} = 0$ to get accurate points, then sketch graph the function, highlight when function is greater than zero and then describe the interval(s). Since a periodic function repeats you would have to account for this.