

5.12 – Trigonometric Functions Review

Radians are a unit of measure used in a circle to relate the sector angle to the sector arc length. It is derived from the fact that the circumference of a circle is related to the radius of the circle by an exact ratio symbolized by π .

Arc length is the term given to this partial circumference measured between two radii.

A **sector angle** is measured between two radii that make an arc.

Ex. formula to relate arc and angle $\theta = \frac{a}{r}$ where $\theta =$ angle in radians
 $a =$ arc length
 $r =$ radius of circle

Ex. leads to conversion formula $360^\circ = 2\pi \text{ rad}$ or $180^\circ = \pi \text{ rad}$

Using the **UNIT CIRCLE** one can come up with exact ratios for the common angles.

x°	0°	30°	45°	60°	90°
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin x	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos x	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan x	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-

More over the Unit Circle allows one to develop the basic trigonometric relationships or identities.

Co-function relationship $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ and $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

Supplemental relationship $\sin \theta^\circ = \sin(\pi - \theta^\circ)$ and $\cos \theta^\circ = -\cos(\pi - \theta^\circ)$

Positional relationship $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$

Compound formulas $\cos(x + y) = \cos x \cos y - \sin x \sin y$ $\sin(x + y) = \sin x \cos y + \cos x \sin y$
 $\cos(x - y) = \cos x \cos y + \sin x \sin y$ $\sin(x - y) = \sin x \cos y - \cos x \sin y$

Double angle formulas $\sin(2x) = 2 \sin x \cos x$ $\cos(2x) = \cos^2 x - \sin^2 x$
 $= 2 \cos^2 x - 1$ $\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$
 $= 1 - 2 \sin^2 x$

Quotient Identities $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta}$

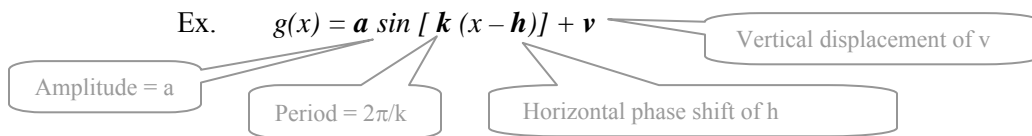
Reciprocal identities $\csc \theta = \frac{1}{\sin \theta}$ $\sec \theta = \frac{1}{\cos \theta}$ $\cot \theta = \frac{1}{\tan \theta}$

Pythagorean identities $\cos^2 x + \sin^2 x = 1$ or $\cos^2 x = 1 - \sin^2 x$ or $\sin^2 x = 1 - \cos^2 x$

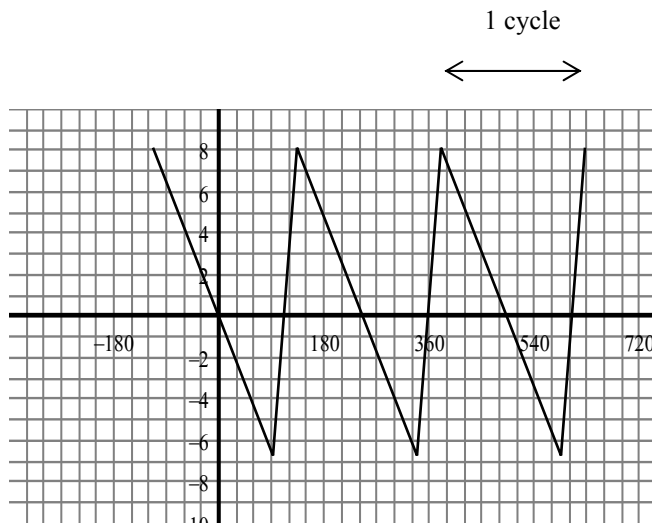
Steps for solving trigonometric equations are;

- re-arrange to isolate unknown
- factor if necessary
- use zero principle in necessary
- use inverse trigonometric function to get angle (deg or rad)
- adjust answer to suit interval using CAST or sketches
- check answer by substituting back into original equation
- Some equations will require one to graph corresponding functions and examine area where the two curves intersect.

To sketch a trigonometric function it is best to use transformations. With “trig” functions instead of using translation and stretch one uses; phase shift, displacement, period and amplitude.



- Cycle** describes one complete pattern (end up at the same y-value that started with)
- Period** describes the horizontal length (x-length) of one cycle
- Amplitude** is half the distance between the maximum (peak) and minimum (trough) values of the function. **Amplitude = (max-min)/2**
- Average** is the middle between max and min of the function. **Average = Min + Amplitude**
- Displacement** is the vertical distance that the median (i.e. average) of the curve has been moved up or down
- Phase shift** is the horizontal distance the curve has been moved relative to its original position



You will need to divide 360° or 2π by period to find “k” factor

Period = $610 - 390 = 420$

Amplitude = $(8 - -7)/2 = 7.5$

Average = $-7 + 7.5 = 0.5$

Displacement = $+0.5$

Phase shift = $+90$

$y = 7.5 f [(360/420)(x+90)] + 0.5$

We don't know the function here, so just use general case.

5.12 – Trigonometric Functions Review Practice Questions

1. Sketch 1 complete cycle for each of the following.

a) $y = -3 \cos\left(x - \frac{\pi}{2}\right) + 1$ b) $g(x) = 2 \csc x - 1$ c) $f(x) = \tan\left(\frac{1}{2}x - \pi\right)$ d) $y = \cos\left(3x + \frac{\pi}{2}\right) + 4$

2. Accurately sketch two complete cycles for $y = 3 \cos(2x - \pi) + 3$

- Describe displacement, phase shift, period and amplitude.
- Describe intervals where the function is increasing
- Describe intervals where the function is stationary (neither increasing or decreasing)
- Calculate average rate of change in the function from i) $-\pi$ to π ii) π to $3\pi/2$

3. Convert to exact radians.

a) 20° b) 1200° c) -90° d) 15°

4. Convert to degrees.

a) 0.45 rad b) $\pi/3$ rad c) $2\pi/3$ rad d) $\pi/4$ rad

5. Calculate the arc length given a radius of 12cm and sector angle of 75°

6. Calculate sector angle given a radius of 12cm and arc length of 10cm

7. If an 70cm door rotates open through 2 radians, what arc distance did the outer edge of the door travel. What area of floor does the door swing over?

8. Expand the following to express each as values of trigonometric functions of one number.

a) $\cos(2x - 1)$ b) $\sin(5 - x)$ c) $\cos(3x + 5y)$

9. Use compound formula to express each of the following as a single trigonometric functions.

a) $\cos x \cos x - \sin x \sin x$ b) $\sin 5a \cos 2a + \cos 5a \sin 2a$

10. Given $\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$ Find $\cos \frac{5\pi}{12}$ as exact value

11. Given $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ Find $\sin \frac{\pi}{12}$ as exact value

12. If $\cos \theta = -\frac{5}{13}$ where $\frac{\pi}{2} < \theta < \pi$ find a) $\sin 2\theta$ b) $\cos 2\theta$

13. Prove the following identities

a) $\cos\theta \tan\theta = \sin\theta$

b) $\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} = \frac{1}{\sin^2 x \cos^2 x}$

c) $\cos\theta(\sec\theta - 1) = 1 - \cos\theta$

d) $1 - 2\cos^2 x = \sin^4 x - \cos^4 x$

14. Find all solutions on interval $\pi \leq x \leq 2\pi$

a) $3\sin x = 2\cos^2 x$

b) $3\sin^2 x - \sin x - 2 = 0$

c) $\cos x = 2\sin x \cos x$

d) $8\sin^2 x - 10\cos x - 11 = 0$

e) $1 - \tan\left(2x + \frac{\pi}{2}\right) = 0$

f) $\sin x \cos \frac{\pi}{6} + \sin \frac{\pi}{6} \cos x = \frac{1}{\sqrt{2}}$

15. When a projectile leaves a starting point at an angle of elevation of θ with a velocity of v , the horizontal distance it travels is determined by the function, $d(v) = \frac{v^2}{32} \sin\theta$, where d is measured in feet and v in feet per second. If an outfielder throws a ball at a speed of 75 miles per hour to the catcher who is 200 feet away, as what angle of elevation was the ball thrown. (1 mile = 5280 feet)

16. The average depth of water in a bay is 4m. At low tide the level is 2m. One cycle is completed every 12 h and low tide last occurred at 15:00h. Calculate the next time the height of the tide will be 5.0m.

17. The population size, P , of owls (predatory) in a certain region can be modeled by the function $P(t) = 1000 + 50 \sin(t - \pi/12)$, where t represents the time in months. The population size, p , of mice (prey) in the same region is given by $p(t) = 20000 + 2000 \cos(t + \pi/12)$. Sketch the graph of these functions on the same grid, and discuss the relationship between predator and prey. Determine the time, to nearest tenth of a month, during the first two years when the populations of the owls is exactly 20 time the population of the mice. You will need to use graphing calculator to help solve this question. Note that equations are in radians.

Answers 1. check answers on calculator 2. a) $y=3\cos[2(x-\pi/2)]+3$ gives $D=+3$, $PS=+\pi/2$, $P=\pi$, $A=3$ b) $0 < x < \pi/2$, $\pi < x < 3\pi/2$ c) $\pi/2, 3\pi/2$ d) i) 0 ii) $12/\pi$ 3. a) $\pi/9$ b) $20\pi/3$ c) $-\pi/2$ d) $\pi/12$ 4. a) 26° b) 60° c) 120° d) 45° 5. 15.7cm 6. 0.83rad or 47° 7. 140cm 8. a) $\cos 2x \cos 1 + \sin 2x \sin 1$ b) $\sin 5 \cos x - \cos 5 \sin x$ c) $\cos 3x \sin 5y - \sin 3x \cos 5y$ 9. a) $\cos 2x$ b) $\sin 3a$ 10. $\frac{\sqrt{6}-\sqrt{2}}{4}$ 11. $\frac{\sqrt{3}-1}{2\sqrt{2}}$ 12. a) $-120/169$ b) $-119/169$ 13. all identities hold true 14. a) no solution on specified interval b) $222^\circ, 318^\circ$ (although question specifies radian, as no exact answer is evident it makes more sense to use degrees for this one. c) $3\pi/2$ d) $4\pi/3, 5\pi/3$ e) $x=11\pi/8, 15\pi/8$ f) $x=\pi/12, 7\pi/12$ 15. 32° 16. 19:00h 17. $t=1, 7.3, 13.5, 19.7$ & $t=4.1, 10.3, 16.6, 22.8$ (solve $\sin(t-\pi/12)=2\cos(t+\pi/12)$ using calculator - see graph below)

Figure 1: Owls & Mice System

