

## 5.11B – Trigonometric Equations

**Example 1:** Solve the following on the interval  $0 \leq x \leq 2\pi$

a)  $\sec^2 x + 5 \tan x = -2$

Identity could be worked out the long way if needed

$$\sec^2 x + 5 \tan x + 2 = 0$$

$$(1 + \tan^2 x) + 5 \tan x + 2 = 0$$

$$\tan^2 x + 5 \tan x + 3 = 0$$

As no integral factors are apparent, turn to quadratic formula.  $a=1, b=5, c=3$

$$\tan x = \frac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)}$$

$$\tan x = -0.6972 \quad \text{or} \quad \tan x = -4.3028$$

Calculator gives  $x = -35^\circ$  or ?

Adjust to interval gives  $x = 145^\circ, 325^\circ$  and ?

Convert to radians to get  $x = 2.5, 5.7, 1.8, \text{ or } 4.9$

As have decimal answers, converting to radians does not make much sense.

Question d should yield same answer as c, because double angle identity was used to re-write it

c)  $\sin 2x = 0$

$$2x = 0 \quad \text{or} \quad 2x = \pi \quad \text{or} \quad 2x = 2\pi$$

$$x = 0 \quad \text{or} \quad x = \pi/2 \quad \text{or} \quad x = \pi$$

Sin function is period and hits zero every  $\pi$  rads N is just used to represent an integral multiplier

can write:  $2x = 0 + n\pi$

so get:  $2x = 0, \pi, 2\pi, 3\pi, 4\pi, \dots n\pi$

b)  $2 \cos\left(x - \frac{\pi}{6}\right) - 1 = 0$

$$2 \cos\left(x - \frac{\pi}{6}\right) = 1$$

$$\cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$$

or if let  $\theta = \left(x - \frac{\pi}{6}\right)$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

so then

$$\left(x - \frac{\pi}{6}\right) = \frac{\pi}{3}$$

$$x = \frac{\pi}{3} + \frac{\pi}{6}$$

$$x = \frac{2\pi}{6} + \frac{\pi}{6}$$

$$x = \frac{\pi}{2}$$

$$\therefore x = \pi/2 \text{ or } 11\pi/6$$

$5\pi/3$  is also a solution for this interval.

Follow the same procedure to the other answer.

d)  $2 \sin x \cos x = 0$

$$\sin x \cos x = 0$$

$$\sin x = 0 \quad \text{or} \quad \cos x = 0$$

$$x = 0, \pi, 2\pi \quad \text{or} \quad x = \pi/2, 3\pi/2$$

Looks like we get extra answers here?

and then consider final answers that are in the specified interval.

Or in degrees would see that sine function hits zero every  $180^\circ$  so  $2x = 0^\circ + n(180^\circ)$

### 5.11B – Trigonometric Equations Practice Questions

1. Find all solutions on interval  $\pi \leq x \leq 2\pi$

a)  $4\sin^2 x = 1$

b)  $2\cos^2 x - 1 = 0$

c)  $\cos x = 2\sin x \cos x$

d)  $8\sin^2 x - 10\cos x - 11 = 0$

e)  $1 - \tan\left(2x + \frac{\pi}{2}\right) = 0$

f)  $\sin x \cos \frac{\pi}{6} + \sin \frac{\pi}{6} \cos x = \frac{1}{\sqrt{2}}$

Use identity

2. What is wrong with the following solution?  $\sin x \tan x = \sin x$

$$\sin x \tan x = \sin x$$

$$\tan x = 1$$

$$x = \frac{\pi}{4} \quad \text{or} \quad x = \frac{5\pi}{4}$$

3. Compare the solutions to  $\sin(x) = 1$  and  $\cos(x - \pi/2) = 1$ . Explain.

4. Compare the solutions to  $\sin(-x) = 1$  and  $-\sin(x) = 1$ . Explain.

5. The index of refraction is a ratio that compares the speed ( $v_a/v_b$ ) of light within two mediums. It can be determined by comparing the angle which the light makes, with respect to the

normal, when passing from one medium to the other. The formula,  $\frac{\sin \theta_a}{\sin \theta_b} = \frac{v_a}{v_b}$ , defines this

relationship. Given the index of refraction of light passing from air to water is 1.33, and the angle of incidence (i.e. angle light makes to the normal from air side) is  $38^\circ$ , calculate the angle of refraction (i.e. angle light make to the normal in the water). By the way, the difference in this angle is why your arm seems to bend when you put it under water.

6. When a projectile leaves a starting point at an angle of elevation of  $\theta$  with a velocity of  $v$ , the horizontal distance it travels is determined by the function,  $d(v) = \frac{v^2}{32} \sin \theta$ , where  $d$  is measured in feet and  $v$  in feet per second. If an outfielder throws a ball at a speed of 75 miles per hour to the catcher who is 200 feet away, as what angle of elevation was the ball thrown. (1 mile = 5280 feet)

7. The function  $h(t) = 4\sin^2(8\pi t)$  describes the height, in meters, of a bouncing ball where the time,  $t$ , is measured in seconds. Calculate when the ball is a height of 2m.

8. The average depth of water in a bay is 4m. At low tide the level is 2m. One cycle is completed every 12 h and low tide last occurred at 15:00h. Calculate the next time the height of the tide will be 5.5m.

9. A weight hanging from a spring is set into motion moving up and down. Its distance  $d$ , in centimeters above or below its “rest” position is described by,  $d(t) = 5(\sin 6t - 4 \cos 6t)$ . At what times during the first 2 seconds is the weight at the rest position (i.e.  $d = 0$ )? This equation is in radians. You will have to use calculator to solve this one. Why?

Switch MODE to radians.

Use CALC and intersect to find points

**Answers 1.** a) no solution in interval specified b)  $5\pi/4$  c)  $3\pi/2$  d)  $4\pi/3, 11\pi/9$  e)  $x=11\pi/8, 15\pi/8$  f)  $x=\pi/12, 7\pi/12$

2. You are dividing out one source of your roots. This would be similar to solving  $x^2=x$  by dividing out  $x$  to left with  $x=1$ . Instead you need to rearrange and factor to  $x^2-x=0$  then  $x(x-1)=0$  to arrive at two roots  $x=0$  and  $x=1$ . In the same way you need to rearrange and factor the trigonometric equation so that you do not miss any roots. 3.  $\pi/2$  solution for both as sine and cosine are complimentary 4.  $-\pi/2$  for both 5.  $27.6^\circ$  6.  $32^\circ$  7.  $t=1/32s$  8. solving  $h(t)=-2\cos(30t-90)+4$  at 5.5 gives  $t=19.62$  or  $19:37$  9.  $t=0.22, 0.75, 1.27, 1.79$