6.1 – Revisiting Function Concepts

Functions – are special relations in which each element of the domain (x-value) corresponds to only one element of the range (y-value). The idea is that each input value (x) will return only one unique output value (y). This is readily evident when graphed as no two points can be vertically on top of each other. A function will pass the Vertical Line Test (VLT)

Ex. The cost of a movie is based on your age. Two people of the same age cannot be charged a different amount. So tell one your age (input) and one can uniquely calculate the cost.

Function notation first used by Bernoulli (1718) differs slightly from the formula notation and provides a useful tool when evaluating a function, considering transformations and examining the results of combining functions. We have studied various functions throughout the course, each of which has it place in helping to model real life scenarios.

Ex. Transformation on $y = f(x)$ function: $g(x) = \boldsymbol{a}\, f\, \boldsymbol{k}\{\,(x - \boldsymbol{h})\} + \boldsymbol{v}$

Stretch horizontally by 1/k

Translate vertically

Stretch and reflection vertically by a

Translate horizontally

Remember that transformations (i.e. multiplying or adding) outside the function will affect the vertical one is adding to the output value (y) after applying the function.

Transformations inside the function affect the horizontal as one is changing the input (x) value before applying the function.

Polynomial functions – a basic type of function with some prime examples being the linear, quadratic, cubic, or quartic. Factoring exposes roots/zeros which allows one to sketch these functions quickly. Transformations can also be used but rearranging the equation to this form may prove difficult for any polynomials higher than degree 2.

Reciprocal functions – this functions puts 1 over a given function. It might be 1/polynomial function or 1/trigonometric function, or 1/?. Reciprocal graphing techniques can be used when function are written in the 1/? form.

Rational functions – same as reciprocal but numerator can now be an expression and not just 1. Detailed analysis (asymptotes, intercepts, test values) are necessary for sketch a graph.

Inverse functions – this function undoes a function and thus can take many forms. For instance, the square root function undoes the quadratic. The logarithmic function is the inverse of the

Exponential functions – raises a base to some exponent.

Logarithmic functions – determines the exponent that a base has been raised to. It is the inverse of the exponential function

Trigonometric functions – best used for periodic behavior. Specific terms; amplitude, displacement, phase shift and period are used instead of translation, stretch and reflect.

The following table outlines some real life examples of the functions we studied and how one might be sketch such a function to get a picture of the scenario.

Function	Application	Equation	Graphing technique
Polynomial	Effect of gravity on objects	H(t) = $-4.9t^2$	Zeros
Reciprocal	Depreciation Certain parts of a piecewise function	Y = csc x	Graph denominator, determine asymptotes and then flip parts of graph section by section
Rational	Business costing calculations	AP(x) = R(x)/x	Asymptotes, intercepts and test values Combinations?
Exponential	Population growth	P(t) = 2^t	Transformations
Logarithmic	Decibel scale	dB = 10 log I/I_o	Transformations
Trigonometric	Ocean tides	H(t) = sin 30t	Transformations
Piecewise		$m(x) = \left\{ \begin{matrix} 2x+1, x<0 \\ x^2+2, x \ge 0 \end{matrix} \right\}$	Graph each part individually

Example 1: Sketch a piecewise function to describe the following scenario

John rides his bicycle at a constant cruising speed along a flat road. He then decelerates (i.e., decreases speed) as he climbs a hill. At the top, he accelerates (i.e., increases speed) on a flat road back to his constant cruising speed, and he then accelerates down a hill. Finally, he comes to another hill and glides to a stop as he starts to climb. Sketch a graph of John's speed versus time and a graph of his distance travelled versus time.

Figure 1: Speed vs. Time

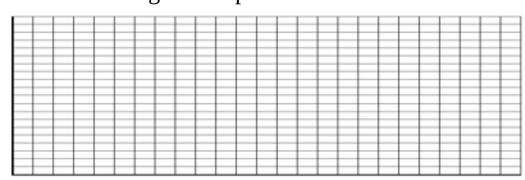
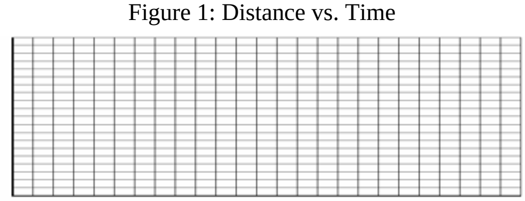


Figure 1: Distance vs. Time



Example 2: Evaluated the following, given;

$f(x) = 2x - 3$ $\qquad g(x) = (x-3)^2 + 1$ $\qquad m(x) = \dfrac{x-1}{2}$

a) $f(3) = ?$

$$f(3) = 2(3) - 3$$
$$= 3$$

b) $g(-2) = ?$

$$g(-2) = [(-2)-3]^2 + 1$$
$$= (-5)^2 + 1$$
$$= 26$$

c) $g \circ f(1) = ?$

$$g \circ f(1) = g[f(1)]$$
$$= g[2(1) - 3]$$
$$= g[-1]$$
$$= [(-1)-3]^2 + 1$$
$$= [-4]^2 + 1$$
$$= 17$$

d) $m^{-1}(3) = ?$

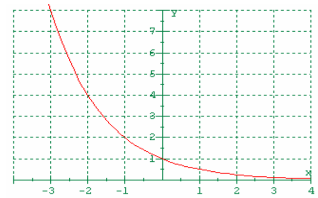
Find inverse function first

$$x = \frac{y-1}{2}$$
$$2x + 1 = y$$

$$so\ m^{-1}(x) = 2x + 1$$

$$m^{-1}(3) = 2(3) + 1$$
$$= 7$$

6.1 – Revisiting Function Concepts Practice Questions

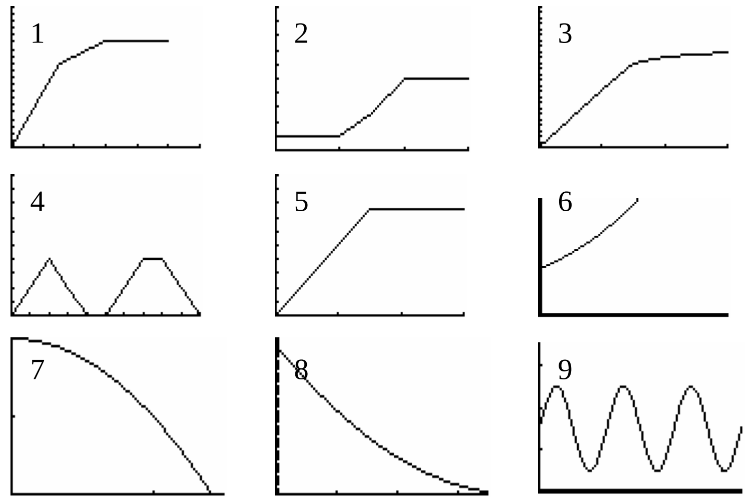


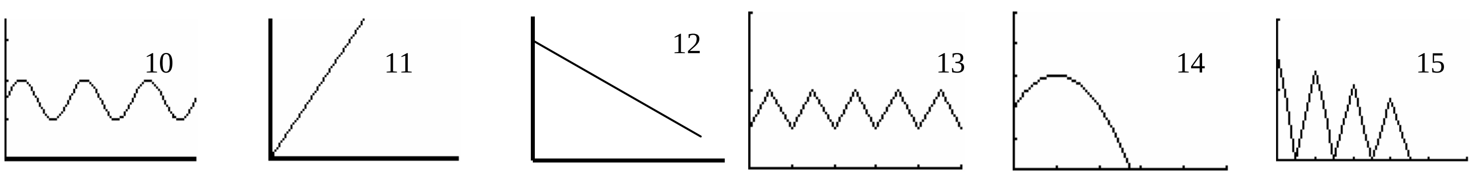
1. Answer the following statements with agree or disagree.

   a) The domain and range of all linear functions is [-∞, +∞]
   b) To find the zeros of a function one can always set the y = 0 and solve for x.
   c) Quadratic and trigonometric functions have max and min values but linear functions don't.
   d) All exponential functions appear as graph show above.
   e) The domain and range of reciprocal function y = 1/x is (-∞,0) and (0,∞)

2. Match the following graphs with the corresponding descriptions. Then suggest what type of relationship might best describe this? (i.e. poly, rational, expo, log, trig, ?)

   a) Walking, then resting
   b) Running, then walking
   c) Resting, walking, resting
   d) Running, walking, resting
   e) Height on a swing
   f) Speed of a baseball
   g) Speed of car through city streets
   h) Height on a seesaw
   i) Height of a yo-yo
   j) Height of a bouncing ball
   k) Path of water from a garden hose
   l) Value of $100 compounded yearly
   m) Value of $100 giving simple interest
   n) The value of a new car over time
   o) The volume of a cylinder as height decreases





3. Suggest what type of function might best model the below scenarios.

   a) The change in value of a new car over time.
   b) The change in value of a new house over time
   c) The pressure in a leaking tire.
   d) The height of a ball thrown in the air

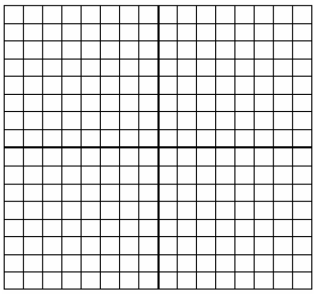
4. Given the functions listed below, write a simplified expression for the following;

$$f(x) = 2x - 3 \qquad g(x) = (x-3)^2 + 1 \qquad m(x) = \frac{x-1}{2}$$

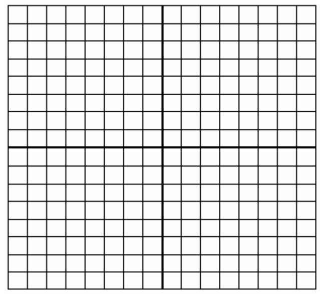
a) f(-5)	b) g(2)	c) m(3)	d) *g o f (2)*
e) *f o g (1)*	f) $m^{-1}(7)$	g) *f o g o m (5)*	h) $g^{-1}[f(m(6))]$

5. Sketch each of the following

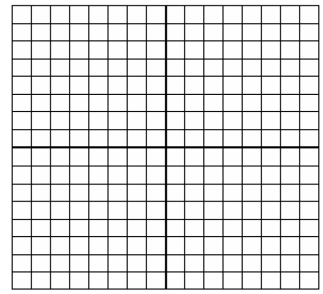
$y=\sqrt{x-2}+1$



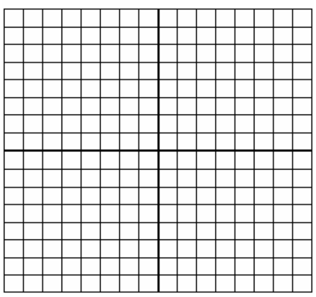
$y=\frac{-1}{x+1}+5$



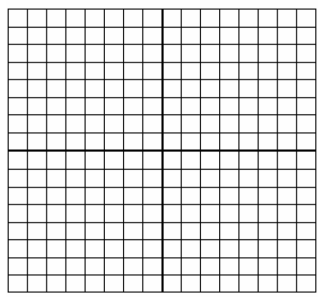
$y=-(x-3)^2+2$



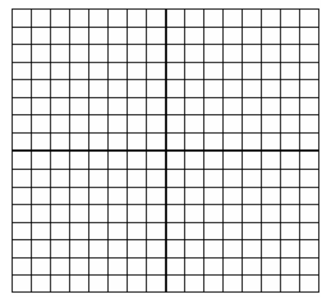
$y=2(x-1)^2(x+4)$



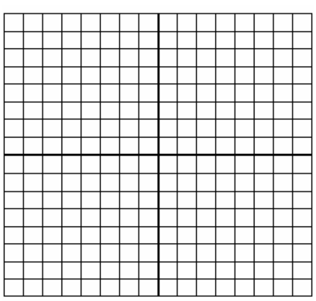
$y=\frac{3}{x+2}-1$



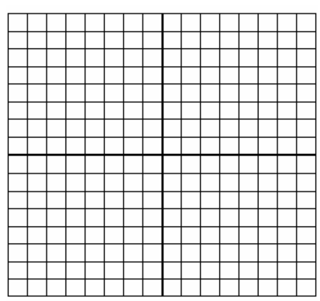
$y=4^{x-1}+3$



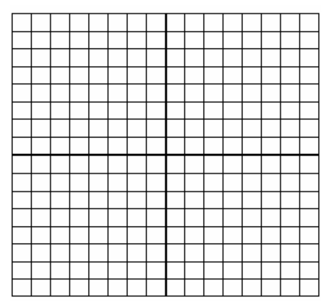
$y=\log_3(x+1)-2$



$y=3\sin 4x-2$



$y=\frac{1}{(x-1)^2(x+3)}$



Answers 1.a) F **b)** T **c)** T **d)** F **e)** T **2.a)** 5 **b)** 3 **c)** 2 **d)** 1 **e)** 9 **f)** 7 **g)** 4 **h)** 13 **i)** 10 **j)** 15 **k)** 14 **l)** 6 **m)** 11 **n)** 8 **o)** 12
3.a) rational or exponential (b>1) **b)** linear or exponential (b>1) **c)** exponential (0<b<1) **d)** quadratic
4. a) -13 **b)** 2 **c)** 1 **d)** 5 **e)** 7 **f)** 15 **g)** 1 **h)** 4 **5.** check on graphing calculator